

Part II

Chapter 3

Federal Public Debt risk management

by Anderson Caputo Silva
Rodrigo Cabral
William Baghdassarian

1 Introduction

Risk management has long been established as an essential activity in financial markets. However, its relevance and sophistication have increased substantially in recent years, mainly due to the expansion of the derivatives market, the greater availability of user-friendly risk management tools, stricter prudential rules like capital requirements and Value at Risk limits, and risk monitoring activities imposed by capital market regulators and central banks.

Efforts to introduce modern risk management practices have also been priorities for Public Debt (PD) managers.¹ Following a series of crises in debt markets in the late 1990s, a growing number of countries explicitly incorporated risk management into their formal debt management objectives, which many define as “minimizing long-term financing costs subject to prudent risk levels”.²

As a result, many Debt Management Offices (DMOs)³ in the world have made significant changes to meet the demand for improved human and technological capital.⁴ In this regard, the most noted change has involved strengthening middle-office capacities, specifically within the risk management and long-term planning areas.⁵ Thus, managing public debt risk has become a fundamental role among DMO functions.

Following this trend, in 2001 Brazil’s National Treasury started a program with the World Bank to build technical capacity and risk management tools and systems.⁶ Two years later, that framework was considered adequate by experts from different countries and international organizations.⁷

Since then, several studies have been produced by the National Treasury risk management team, presented in academic and professional seminars and published⁸ in different milieus. These studies are key to the ongoing

¹ In this chapter, we will use the terms public debt risk manager and risk managers to represent public debt managers responsible for risk management.

² See *Guidelines for public debt management* (2001).

³ The generic term DMO (debt management office), already consolidated in the literature, is used to designate the government area/unit in charge of this function.

⁴ Some examples are the UK, France, Germany, Brazil and more recently Mexico.

⁵ See Organisation for Economic Co-operation and Development (OECD) *Studies in risk management* (2006).

⁶ International experts from academia and financial markets, as well as a firm specialized in developing IT systems for financial markets were hired to help build Brazil’s risk management framework.

⁷ The workshop on Public Debt Management in Brazil was held in Rio de Janeiro at the Getulio Vargas Foundation (FGV) in March 2003. Participants included senior public debt management experts from nine countries – Brazil, Denmark, Spain, the US, UK, France, Italy, Portugal, and Czech Republic, as well as representatives of the OECD and World Bank.

⁸ References to more technical studies are cited throughout the chapter.

effort to improve risk management practices in Brazil. However, the highly technical character of the subject may create problems to understanding each aspect and how it fits with the others to form the complete set of public debt risk managers' responsibilities.

This chapter describes the scope of activities and challenges in managing public debt risks. In addition to providing an overview of how the National Treasury addresses risk management, it also intends (ambitiously) to meet or minimize recurring demands from researchers and countries at the initial stage of capacity building for a consistent set of tools and responsibilities. Further, a thorough view of the tools that need to be developed and the skills required could be a useful guide for those interested in improving their risk management practices.

The chapter is organized as follows. Following this Introduction, Section 2 offers an overview of the scope of PD risk managers' activities, dividing their duties into Public Debt Risk Management (PDM) peripheral and main functions. Section 3 features the main risk indicators used by a DMO. Section 4 highlights the important role of risk managers in providing, based on qualitative analyses, a long-term benchmark to guide short and medium-term debt strategies as well as their role in designing and monitoring debt strategies, followed by a brief discussion about assets and liabilities management (ALM). Section 5 discusses the peripheral duties of public debt managers and Section 6 offers some conclusions.

2 Scope and main challenges in public debt risk management

This section describes the wide array of tasks risk managers must perform, as well as the major challenges they face when meeting the demands of different clients (typically government authorities or senior technical staff) and counterparts (debt managers). Most of the challenges are related to adjusting risk management tools already used by scholars, investors and market analysts to the specific needs of a public entity holding a net liability portfolio.

The list of a risk manager's duties is long and open to debate - as to which functions should be included. Nonetheless, they can be grouped in two categories: debt management main and peripheral functions.

Main functions include identifying long-term benchmarks (optimal debt composition), developing and regularly assessing risk indicators (to measure several types of risks) and designing, monitoring and analyzing trade-offs across different refinancing strategies that can be implemented by the DMO.

The most common tasks with respect to peripheral functions, which are not the exclusive domain of public debt risk managers, include debt dynamics' exercises and sustainability assessments. Debt dynamics and sustainability tests are conducted by an array of interested parties such as financial analysts, scholars and fiscal policy makers. However, due to their access to private knowledge of the government refinancing strategy, risk managers are in a privileged position to conduct debt dynamics and sustainability exercises. That information allows them to produce a more precise debt dynamics forecast with a better evaluation of the risks involved (see Section 5 for more details).

Risk managers are also responsible for providing relevant inputs for prudent debt management - which include general guidelines for the optimal composition of debt and a broad set of risk indicators - as well as elaborating, supervising and assessing the trade-offs of different refinancing strategies.

A fundamental question for PD managers is "what debt profile and structure should the government pursue?" In other words, "what is the optimal long-term debt structure?" The public debt risk manager plays an important role in addressing this question by pointing out the trade-offs regarding costs and risks of different long-term debt strategies.

Debt management theory sheds some light on the general characteristics of the debt portfolio. However, in order to provide specific, quantitative guidelines, several debt specialists have engaged in a debate about the ways in which benchmarks are determined that has gained increased attention in DMOs around the world.

Another important function of PD risk managers is their active participation in the design, monitoring and analysis of trade-offs across different refinancing strategies that can be implemented by DMOs. The process of designing a debt strategy - which was explored in the previous chapter - is in many ways a shared responsibility with other DMO areas, such as the front-offices.

Risk managers identify possible risks for carrying out the debt strategy and refine the estimates (targets) for the future debt structure and profile (typically one year, as is common in annual borrowing plans). They also monitor the implementation of a debt strategy to validate and suggest corrective measures where needed, with respect to the issuance strategy.

Section 4 explores the optimal structure for the Brazilian debt, presents the analytical model currently used by the National Treasury, and describes the role of risk managers in developing and monitoring short and medium-term strategies.

One of the debt managers' central functions is to develop and monitor a comprehensive set of risk indicators, which do not necessarily need to be sophisticated. In practice, a set of simple indicators such as profile, average maturity, refixing duration and maturity structure (measured as a percentage of debt maturing in the short-term, for example), can provide useful information on the refinancing and interest-rate risks of the debt.

More sophisticated risk measures, which usually involve stochastic simulations, are increasingly used in DMOs to complement traditional measures. These are mainly adaptations of indicators that have already been developed and are known to investors (such as the Value at Risk - VaR). Thus, the main challenge is to adjust these indicators to the debtor's point of view, especially that of the PD managers. Commonly used indicators are the Cash-Flow-at-Risk (CFaR), the Cost-at-Risk (CaR) (or Stock-at-Risk - SaR) and the Budget-at-Risk (BaR).

Stochastic analyses are also frequently used by risk managers to provide inputs for decisions involving specific transactions. Typical examples are debt exchanges and other liability management operations that require trade-off assessments in terms of cost and risk.

The set of risk indicators also needs to include measures from the demand side. That is, PD managers need to monitor possible sources of discontinuity or failures in the planned debt issuance strategy. These measures are particularly important in emerging markets, where the amount of risk (notably interest-rate risk) that the investor base can bear represents a significant constraint to the smooth implementation of a debt strategy.

Just as market participants measure their interest-rate exposure through indicators such as the Present Value of a Basis Point (PVBP) and VaR, risk managers should perform this task in order to identify the pace and amount of risk transferred from the government to the private sector that a given refinancing strategy entails. In some circumstances, for example, abnormal VaR levels caused by macroeconomic volatility may significantly reduce the demand for fixed-rate securities, forcing debt managers to pay higher costs or even make sudden changes in the profile of their issues. Traditional and stochastic risk indicators, in addition to risk measures on the demand side, are discussed in Section 3.

Finally, in order for DMOs to cope with all these functions, they must invest significantly in human and technological capacity. The development of risk management systems that compare trade-offs in terms of cost and risk across different potential refinancing strategies is an essential step that can substantially improve a DMO's ⁹ decision-making process.

⁹ The National Treasury developed a public debt refinancing and risk management system - the GERIR system. It provides the foundations to front- and middle-office work in the formulation and analysis of debt strategies. Through GERIR, debt analysts simulate

3 Public debt risk indicators

Public debt risk has many dimensions; this section illustrates the main indicators commonly used by PD risk managers. Most use rather simple computations and can be called *traditional indicators*. Others use stochastic simulations and usually belong to the so-called *risk family* indicators or *at-Risk*. Although they are not overly complex, they represent adaptations of measures frequently used by the private sector such as the Value at Risk (VaR), from the debtor's standpoint.

It should be noted that, despite the simplicity of the traditional indicators, many countries do not compute them and there appears to be no international methodological consensus among those that do. The first problem stems from the fact that many DMOs lack back-office systems that can compute in an aggregate and accurate fashion even the simplest debt indicator (i.e., the outstanding debt). The lack of consensus also has important consequences as it makes cross-country comparisons of risk indicators a complex task.

To illustrate this point, Table 1 shows the average maturity of Brazil's public debt using two different methodologies; these generate meaningful differences in the statistics produced, to the extent that not only debt managers, but also a whole range of investors and rating agencies use these indicators for international comparisons.

Table 1. Differences between average maturity and average life

December 2008	Average maturity (traditional methodology)	Average life (alternative methodology)
Federal Public Debt (FPD)	3.5	5.6
Domestic Debt (DFPD)	3.3	4.9
External Debt (EFPD)	5.6	12.3

Source: National Treasury

The traditional methodology (average maturity)¹⁰ used by the National Treasury takes into account all disbursement flows (including coupon payments) when calculating the average public debt term. However, the methodology used by most countries is limited to the flow of principal payments. Although there is no internationally unified nomenclature in this area, these two forms of computing are usually referred to as average term and average life respectively. The first puts greater emphasis on the debt refinancing risk but - being more conservative - makes it difficult to draw international comparisons. For this reason, the National Treasury publishes the statistics according to both methodologies.

With the traditional methodology, the average term of domestic debt was 3.3 years at the end of 2008 while for international comparison purposes, the average life was 4.9 years. This average life exceeded, for example, that of the debts of many countries with credit risk ratings better than Brazil's. Moreover, while most countries use nominal flow values, Brazil computes the present value of each.¹¹ In a debt with a significant number of long-term securities and coupons, these differences are even more striking.

several refinancing strategies for the Brazilian public debt and compare their results, as measured by a set of relevant indicators. The system was developed after an in-depth investigation of international experiences and was evaluated by risk specialists from several countries (see Annex 1 for more details about the system).

¹⁰ The subject, which is also explored in Section I, Chapter 1, is discussed in detail in Annex IV.

¹¹ As more exploited in Annex IV, the methodology considers all flows (interest and principal) and bring them to present value using the original issuance rate of the Bond (not market values).

The risk measures are presented in the following subsections, grouped by the most relevant types of risks¹² to which the debt is exposed - namely, market risk,¹³ refinancing risk, budget risk and demand-side risk.

3.1 Market risk

Market risk can be defined as the uncertainty related to changes in the expected costs arising from the volatility of market variables (interest rates, exchange rates, inflation, etc.). Although in financial markets this type of risk is associated with the volatility of asset prices, in the case of public debt, this type of risk refers to changes in the value of the portfolio (outstanding debt). And, while this is an apparently simple concept, there is a relatively broad discussion involving the methodology used to compute market risk due to differences on the relevant outstanding debt measure, which is the basic element for a market risk measurement. Some frequent questions (to be considered by DMOs) are (a) should the debt value be assessed in terms of mark-to-market, the way banks deal with financial assets, for example, or mark-to-curve, i.e., based on the yield at which each security was originally sold? Also, (b) should the outstanding volume be expressed in nominal or in real values?¹⁴

Many countries do compute market risk measures. In the category of traditional indicators, profile, duration, refixing duration and convexity are the most common, while the so-called Cost-at-Risk stems from the stochastic group of indicators. Stress tests are commonly used as complementary in market risk analysis to measure the consequences of severe shocks, most often in the interest and exchange-rates.

The concepts of duration and convexity are well established in the financial literature and there are generally no significant methodological differences between the way to compute them from an investor's or a risk manager's standpoint.¹⁵ Duration targets are in fact used by many countries, such as Denmark and Sweden. For these reasons, they are not described at length, here.¹⁶

Refixing duration is a less frequently applied concept. It measures the average time it takes for the whole debt to be affected by a change in interest rates. For fixed-rate bonds, this is equivalent to duration. However, for floating-rate bonds, it represents the time span between changes in relevant interest rates (in the Libor, for example). Since the outstanding debt in most countries usually consists of a mix of nominal and indexed instruments (most often, floating-rate, inflation-linked, or exchange-rate linked), the use of this indicator has become increasingly common.

$$Duration_{refixing} = \frac{1}{PV} \times \sum_{i=1}^n PV_i \times T_i \quad (1)$$

¹² There are other types of risks not covered here, such as credit risk and operational risk, which are discussed at the end of the section.

¹³ Although the text describes it as *market risk*, a more precise expression would be *value at risk*, since the outstanding debt or debt value in Brazil is not evaluated in terms of mark-to-market value, but in terms of the present value of the flows and uses the issuance rate of each security for discount purposes.

¹⁴ These and other discussions about the outstanding debt are explored in Baghdassarian (2003) and Bonomo et al. (2002).

¹⁵ In many cases, investors use Mark-to-Market values, while debt managers use Mark-to-Curve.

¹⁶ There is a debate about the use of duration as a target. Care must be taken in following these targets as it may entail odd policy conclusions from a debtor point of view - such as an increase in interest rates that causes a reduction in duration. To follow the target, the DM would be induced to issue longer-term debt, but this is exactly the time that interest-rates are higher!

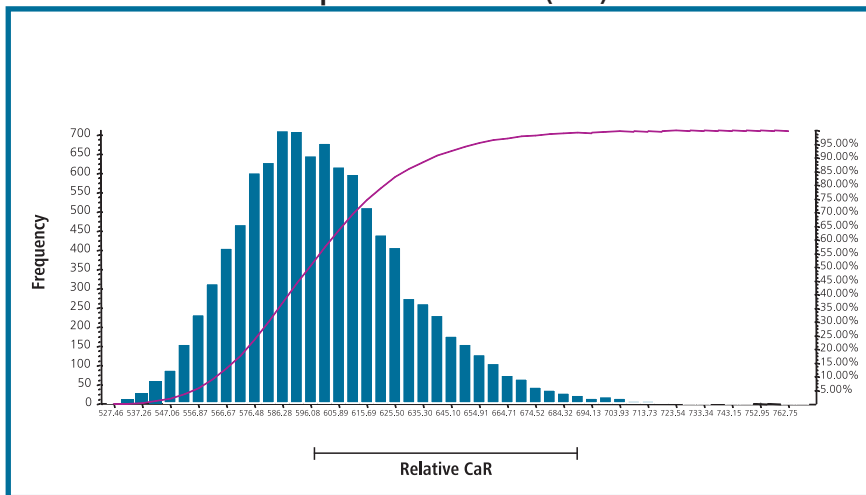
where:

- PV – Present flow value
- $T_i = 0.03$ (one day) – For floating rate securities
- $0 < T_i < 1$ – For TR securities
- $0 < T_i < 3/6$ – For Libor 3/6 months
- $T_i = t_i$ – For others

The Cost-at-Risk-CaR¹⁷ represents the maximum expected value the outstanding debt can reach over a given period, considering a given level of significance. While the market-risk indicators refer to the sensitivity of the debt to sudden changes in market variables - especially in interest rates - the CaR provides a measure of uncertainty about the expected value of the outstanding debt in the future (say, over one, five and 10 years). This indicator also has the advantage of incorporating the effects of a broad range of risk factors that can affect the outstanding debt such as changes in interest rates, inflation, exchange rates and Gross Domestic Product (GDP) (when appropriate), as well as the possible correlation among them.

Both the absolute and relative CaR can be calculated. Absolute CaR is the difference between the future outstanding debt, in view of a given level of significance and its initial value; relative CaR measures the difference between this same maximum value of outstanding debt at a given significance level and the average distribution of future outstanding debt (see Graph 1).

Graph 1. Cost-at-Risk (CaR)



The use of CaR is also related to the role risk managers play in debt sustainability assessments:¹⁸ It is an instrument that aggregates uncertainty as well as hypotheses on refinancing the debt.

Moreover, despite their similarities, there are relevant differences between CaR and the well-known VaR: e.g. the CaR is an adaptation of VaR that takes into account the issuer's specific needs, which involve the value

¹⁷ Baghdassarian (2003) presents the methodologies for *CaR*, *CFaR* and *BaR*.

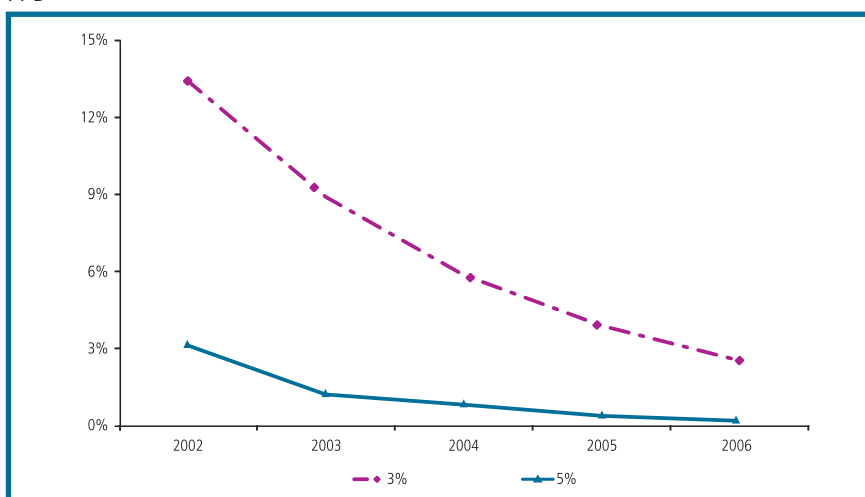
¹⁸ This discussion is explored in Section 5.

of its outstanding debt (often based on mark-to-curve as opposed to mark-to-market in VaR) over a much longer time than the usual for VaR (e.g. one day). These subtle differences, including the importance of considering the refinancing strategy, translate into significant challenges when modeling this tool, especially with regard to pricing the different instruments to be issued in the future (strategy) and their relative risk premia.¹⁹

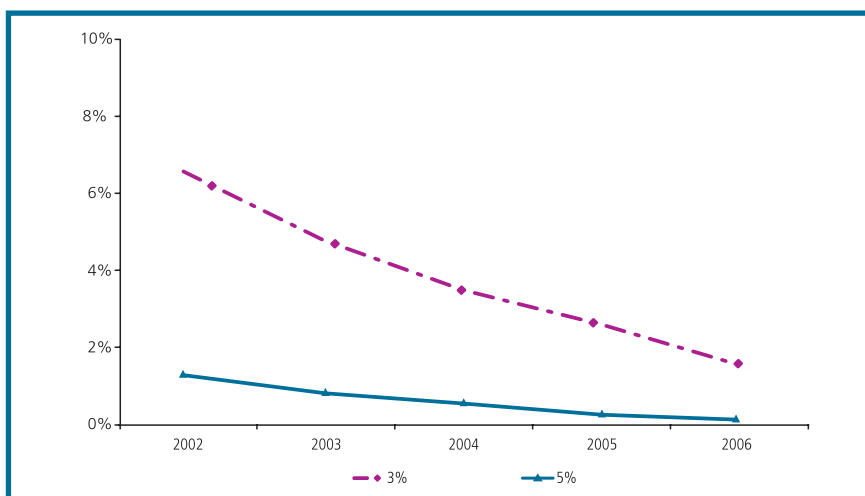
Finally, as a complement to the measures above, risk managers generally conduct stress tests, in several ways. A common approach is to apply shocks to key variables, such as interest and exchange rates, in terms of standard deviations, based on the distribution of their historical values over a period of time. Measuring the sensitivity of these shocks is now common practice in Brazil, either to determine how they behaved in the past or measure the future consequences of the debt strategy being implemented/analyzed. Rating agencies and financial analysts²⁰ have also used the stress tests. Graph 2 provides an example of a stress test performed for the Brazilian debt and included in the 2006 Annual Borrowing Plan.²¹

Graph 2. Probability of 3% and 5% increases in relation to GDP in outstanding debt as a result of shocks in interest and exchange rates

FPD



DFPD



3.2 Refinancing risk

Public debt refinancing risk is defined as the risk of sudden changes in the debt payment structure upon its refinancing. In extreme cases, governments would be unable to roll over part or all debt falling due at a given time.

As with market-risk, refinancing risk indicators can also be divided into traditional ones and probabilistic measures (at-Risk). The National Treasury uses three traditional indicators to assess this type of risk: average maturity, debt maturity structure (especially the debt percentage maturing in the short term) and the Cash-Flow-at-Risk (CFaR). As will be shown below, each indicator measures the refinancing risk from a different perspective and their joint use is recommended.

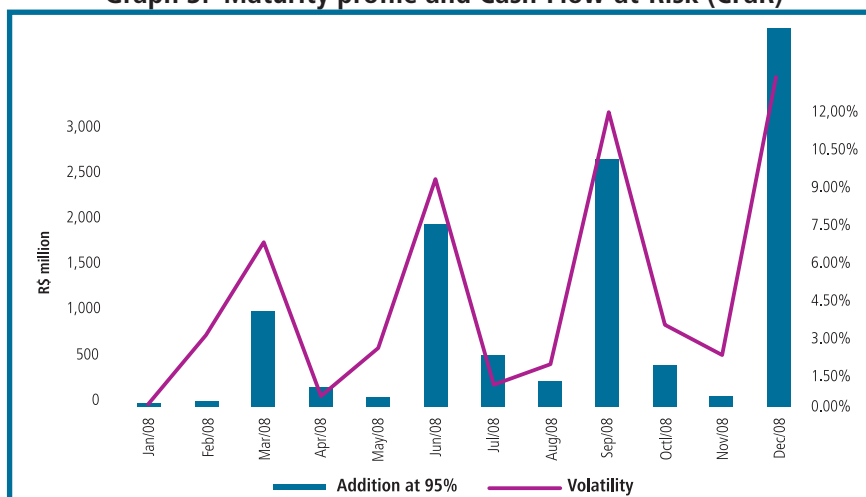
Average maturity measures an equilibrium point of all debt maturities. Since it is an average, it is important to follow the indicator over time, since this might prevent a systematic debt shortening, an undesirable outcome for debt management policies.

The second indicator is the percentage of the debt maturing in 12 months, a complementary measure to the average maturity, and focuses on the short term. While the latter measures systematic reductions in debt flow payments, the former is more focused on cash requirements to pay the debt in that time frame. In other words, it is related to the liquidity risk. A natural generalization is full maturity following, for example, the percentage maturing in one, two or three years, up to five, 10, etc.

The last indicator is the Cash-Flow-at-Risk (CFaR), which measures the uncertainty associated with future cash-flows. CFaR determines, with some level of significance, the maximum expected cash-flow (payments) at specific future dates or periods. (A fixed-rate bond denominated in local currency does not entail this type of risk, since there is no risk factor associated with its cash-flow)²². However, it is hard to determine ex-ante what the cash-flow of a foreign exchange security expressed in local currency will be. The same rationale applies to other types of instruments, such as floating-rate or inflation-linked securities.

The graphs below show how the National Treasury uses this indicator by illustrating the trade-offs across different instruments in terms of cash-flow.²³

Graph 3. Maturity profile and Cash-Flow-at-Risk (CFaR)



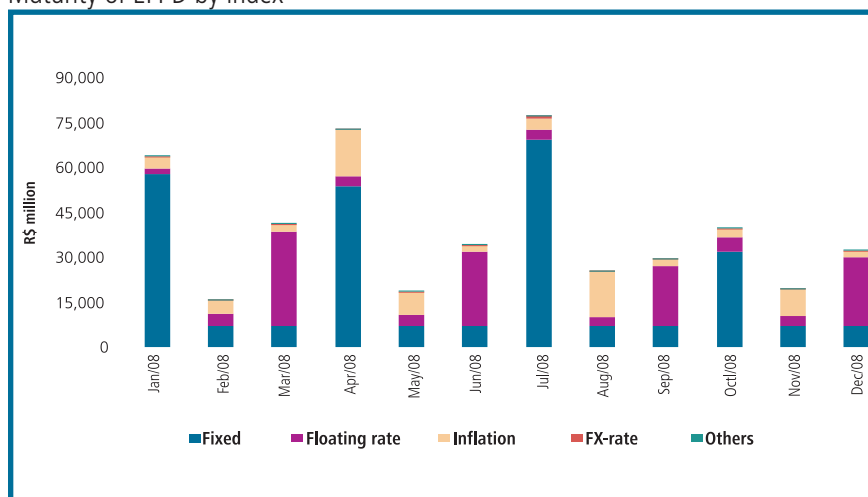
Source: National Treasury

¹⁹ For further discussions on these complexities, see Bonomo, Costa, Rocque and Silva (2003) and Cabral (2004).

²⁰ Rating agencies, such as Fitch, Moody's and Standard & Poor's, have used this type of analysis.

²¹ For a more detailed discussion see the Annual Borrowing Plan (2006).

Maturity of EFPD by index



Source: National Treasure

As noted, the indicators discussed in this section are more complements than substitutes to measuring refinancing risk. While the average maturity and percentage of the debt maturing in the short term are more focused on the time distribution of debt payments, the CFaR focuses on the volume - and its sensitivity to shocks - of payments that the debt manager will have to honor on specific future dates.

3.3 Budget risk

The concept of Budget-at-Risk (BaR) as used in Brazil relates to the risk that the debt service within a fiscal year will exceed the amount originally approved by Congress.

Since debt service in the budget is measured in monetary terms (cash flow), the BaR is very similar to the CFaR in that both measure the uncertainty of cash-flows. The major difference is that while the BaR focuses on the fixed one-year period (fiscal year), the CfaR is more flexible and can be computed for any specific date or period. Moreover, the BaR has an exogenous reference value which is approved by Congress and thus involves the probability that that value might be exceeded. Conversely, the CFaR provides, for a given level of significance (risk), the maximum expected value for the cash-flow on a given date or period.

The close monitoring of budget risk in Brazil, and in other countries in general, is an important task of the PD risk manager. By noting the probability of exceeding the budget, debt managers can anticipate or avoid the potentially hard and time-consuming mission of submitting to Congress a proposal for a supplementary budget to pay the debt. Although one would expect that the request would not be denied, exposing the sovereign debt to such a process could be a sensitive issue that calls for appropriate monitoring.

3.4 Demand-side risk

Demand-side risk is defined as the risk of sudden changes in the demand for government bonds. Although this may occur due to several factors, the most common are interest rates. Investors, due to stricter prudential

²² There is an interesting discussion, but one that will not be explored here, regarding the cash-flow risk-free bond. Most countries consider the fixed bond as such. However, others more concerned with real variables could argue that inflation-linked securities would be the ones that should be considered as risk-free.

²³ For more details on the CFaR computing methodology, see the *Annual Debt Report* (2004).

regulations or domestic investment policies, have increasingly resorted to measures of interest-rate exposure to monitor their loss risks. In fixed-income markets, two of the most common are the Present Value of a Basis Point (PVBP) and the Value-at-Risk (VaR).

The PVBP²⁴ expresses how much the portfolio value would change given a one basis point (0.01%) variation in interest rates. It is similar to the concept of duration, with the advantage of also being a function of the portfolio's total volume.

$$PVBP = P(i) - P(i + 0,01\%) \tag{2}$$

where:

- i – Return rate (yield)
- $P(i)$ – Security price

The Value-at-Risk (VaR) complements the PVBP by incorporating asset price volatility. While the PVBP measures absolute sensitivity to changes in interest rates, the VaR enhances the information set by including the probability of such changes occurring.²⁵

$$\sigma_p^2 = w' \Sigma w \tag{3}$$

where:

- σ_p^2 – Return rate (yield)
- w – Vector of weights for different securities in the portfolio
- Σ – Variance/covariance matrix of R returns on securities in the portfolio

$$VaR = P_0 \cdot \sigma_p \cdot 1,95 \tag{4}$$

where:

- P_0 – Initial price
- 1.95 – Equivalent to a 95% confidence level

A significant share of the demand for government securities, especially in Brazil, is subject to limits concerning exposure to interest-rate risks. This constrains debt managers, who then need to transfer interest-

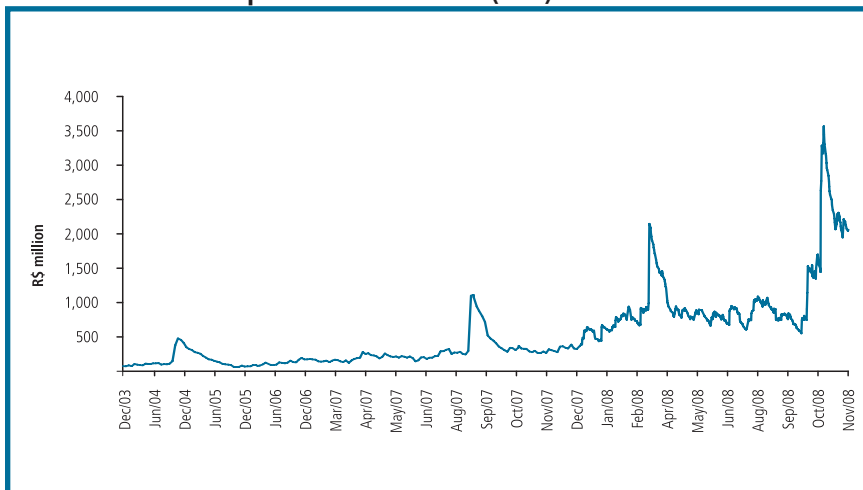
²⁴ Also known as Dollar Value of 1 Basis Point – DV01.

²⁵ It is possible in an economy experiencing a stabilization process, for example, to have an increase in the PVBP, due either to the increased volume or duration of fixed securities (or both) and, at the same time, a decrease in VaR (as a result of a sharper decline in volatility).

rate risk to the market. Worse, in times of volatility, the VaR can reach high levels and lead to investors' stop-loss operations.

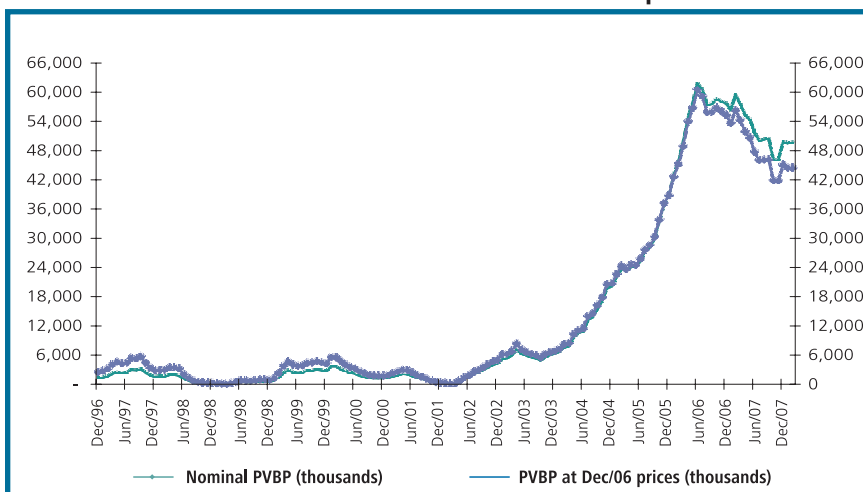
The effect of these changes on demand can be disastrous to a debt strategy. Thus, the risk manager must monitor this risk - not only the risk of the current portfolio, but also the implicit pace of risk transfer that a given debt strategy entails for the future. This follow-up is particularly relevant in countries that are lengthening the maturity of their bonds and increasing the share of fixed-rate instruments. Brazil fits this profile and the National Treasury monitors both indicators, as shown in Graphs 4 and 5.

Graph 4. Value-at-Risk (VaR) evolution



Source: National Treasury

Graph 5. PVBP evolution
PVBP - Nominal and at December 2006 prices



Source: National Treasury

4 The optimal long-term profile (benchmark)

This section highlights one of the PD risk manager's fundamental tasks - i.e. establishing long-term targets to guide the short and medium-term debt strategies.

As a first principle of PDM and according to Ricardian Equivalence assumptions, as defined in Barro (1974), such management would be irrelevant.²⁶ Although the theoretical investigation of the Ricardian Equivalence is undoubtedly useful, there is solid evidence and broad consensus that the strong assumptions behind it do not hold, in practice. These assumptions are: (a) agents have infinite planning horizons (complete information); (b) markets are complete; (c) taxes are non-distorting.

The reasonable relaxation of these assumptions makes risk management relevant. Tax smoothing, market completion, public policy signaling, among others, are some countries' public debt objectives.²⁷

However, once PD management is recognized as a relevant subject, identifying the desirable debt structure in the long term becomes a key question that needs to be answered so as to guide short and medium-term debt operations.

In order to better understand the role of this long-term reference for debt managers, one can draw an analogy to the situation of an adventurer in a forest carrying a compass to know what direction to follow; this may be the only way to be certain the next steps will lead to the desired location. For debt managers, the long-term profile (benchmark) represents the direction they want to follow and the tool (as with the compass) they use to develop and monitor their strategy.

Literature on Debt Management (DM) provides some guidelines on the general characteristics of the debt portfolio. However, in order to provide more specific, quantitative guidelines, several specialists have raised issues that have gained increased attention in DMOs everywhere. Indeed, the search for appropriate methodologies to develop a benchmark has become an important topic on the research agenda of debt managers in several countries, including Brazil, Canada, Denmark, Portugal and Sweden.

The contribution of institutions such as the World Bank and the IMF to this debate has been important. In their joint publication "Guidelines for Public Debt Management" (2001),²⁸ they define the benchmark as a powerful tool to represent the debt structure a government wants to achieve, based on its risk and expected cost preferences.

Usually, the benchmark is represented by a set of relevant debt indicators, such as profile, duration, debt structure, etc. The idea is that it should be a long-term goal, representing social preferences. In mathematical terms, it can be seen as an optimization problem, where a government wants to maximize its utility function in view of some constraints.²⁸

Some countries may decide on their optimal debt structure (benchmark) based on very simple analysis and ad-hoc assumptions. Debt managers could conclude, for example, based on their beliefs about the benefits of diversification, that the ideal debt structure would be a mix of nominal and inflation-linked securities. The rationale would be that a portfolio with such assets can generate a debt-servicing structure more resistant to recurring demand and supply shocks.

²⁶ When discussing long-term goals and debt strategies, it is important to point out that they make sense from a theoretical standpoint.

²⁷ The literature about the relevance of debt management is not restricted to the flexibility of Ricardian Equivalence assumptions. Here, it suffices to justify its relevance. Lopes (2003) and Bonomo et. al. (2003) offer brief summaries of the literature.

²⁸ As most countries define their main DM objective to be minimizing long-term costs subject to prudent risk levels (see IMF and World Bank, 2001), identifying the objective function and constraints is straightforward.

Others could follow the (desired) path of calculating relevant risk indicators for some different (hypothetical) debt profiles and investigating the trade-offs they produce. This would be an efficient way to assess the pros and cons of different portfolios.

Finally, an even more analytical framework could be used that builds a model from which the optimal portfolio would emerge endogenously. However, it is difficult to imagine a supra-rational model capable of considering all the objectives and constraints of public debt management and, by itself, giving the answer to this optimum profile question.

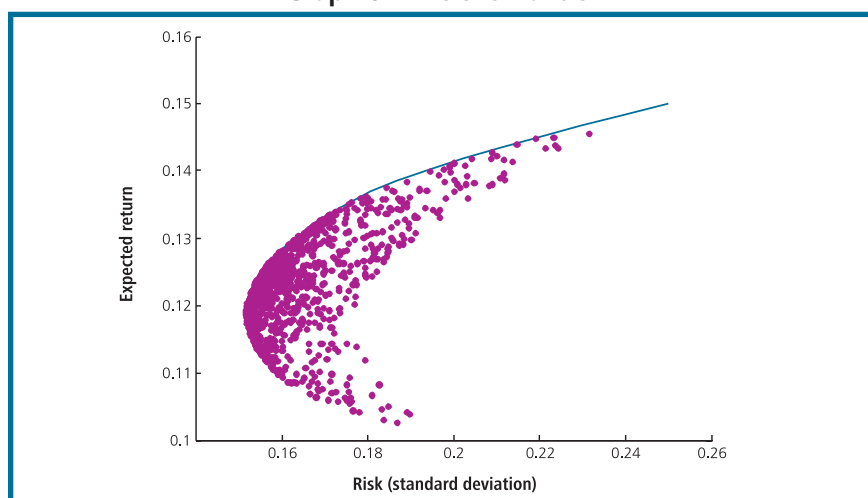
As mentioned before, Canada, Denmark, Portugal, Sweden and Brazil are countries that use more analytical frameworks for benchmarking. Cabral (2004) describes briefly how they address this subject.²⁹

Portugal was one of the first countries to develop such a methodology. Granger (1999) and Matos (2001) illustrate how their model works: Basically, it is a cash-flow simulation model, having as inputs stochastic simulations of interest rates, different financing strategies and deterministic scenarios for other economic variables, resulting in some "efficient" portfolios.

The Swedish model is also based on cash flows, with auto-regressive processes for inflation, GDP, long-term interest rates and exchange rates, as well as a Taylor rule for short-term interest rates. With some assumptions about borrowing requirements, a number of portfolios are evaluated, with nominal and real cost measures. Bergstrom and Holmlung (2000) describe the model in detail.

The Brazilian approach, as described in Cabral and Lopes (2004) is basically an efficient frontier analysis, where costs and risks are measured in terms of debt/GDP ratios. Steady-state compositions are simulated through a number of different periods based on stochastic scenarios and assumptions about the pricing of assets. With some portfolios evaluated in terms of cost and risk, as well as the correlation matrix, it is possible to draw an efficient frontier.³⁰ Then, having observed it, debt managers would choose, based on their attitude towards risk, the point (portfolio) to represent the benchmark.

Graph 6. Efficient frontier

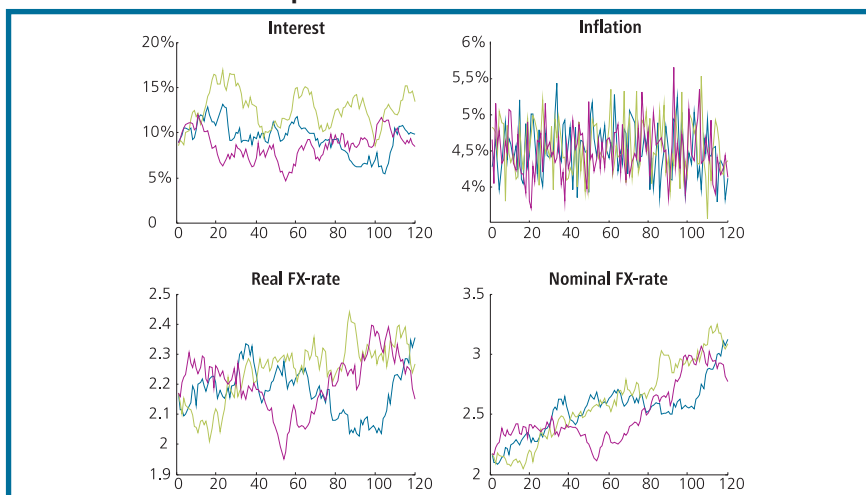


²⁹ Other useful references for international experience are Guidelines for Public Debt Management (2001) and Nars (1997).

³⁰ It is important to note that this is an efficient frontier from the issuer point of view, quite different from the one designed by an investor.

In the Brazilian model, the stochastic scenarios can be generated by two different and somewhat complementary ways. In the first, some financial stochastic models are used, namely a Cox, Ingersoll and Ross (CIR) model for domestic and external interest rates, a Brownian process for inflation and a Chan, Karolyi, Longstaff and Sanders (CKLS) model for real exchange rates. Residuals are correlated using a Cholesky decomposition. The second methodology uses a macro-structural model to describe the evolution of the main economic variables (an IS and Phillips curve, a Taylor rule and equations for exchange rates and risk Premium behavior).³¹

Graph 7. Stochastic simulations



With the above models, it is possible to run Monte Carlo simulations and obtain a distribution of debt/GDP ratios over a specific time horizon for each portfolio (debt profile) considered. The mean and standard deviation are taken from the distributions and used as cost and risk measures and correlations are calculated based on simulation of portfolios with single and mixed assets, which leads to the design of an efficient frontier.

Other issues deserve attention in the process of conducting these exercises. First, a benchmark model should ideally be independent from current market conditions. Although this may seem odd at first, it should be remembered that it is precisely the separation of long-term objectives from circumstantial and tactical constraints that makes the use of the benchmark unique.

Further, benchmark modeling should incorporate, to the extent possible, constraints on the demand side, i.e., if optimum composition is based solely on supply side objectives, and the potential demand for that portfolio is not examined, this is a myopic decision that has a reduced chance of succeeding.

With respect to more general aspects, benchmark formalization is also fundamental. In fact, if a benchmark is established but not formalized,³² it may be useless.³³ Besides, a formalized benchmark affords transparency for risk managers as well as a degree of continuity across different governments. Establishing a well-defined governance process is yet another crucial step, although this matter is outside the scope of this chapter.

³¹ Annex III provides a brief description of the model currently used by the National Treasury. Cabral (2004), Cabral and Lopes (2005) and Costa, Silva and Bagdassarian (2003) contain examples of the model implementation.

³² Formalization is approval by the minister or Congress, which could delegate to the DMO the power and duty to pursue those objectives.

³³ Obviously, the same argument applies to the medium and long-term strategies.

Another critical issue involves the design of a transition (medium and long-term) strategy to the optimal profile (benchmark). This is not an easy task, particularly in less developed countries with debt portfolios that are far from optimal. This transition would involve a complex problem in the attempt to find the strategy that optimizes the path between current conditions and long-term goals. Also, in these countries, it might just happen that the existence of various market constraints would eliminate many potential strategies that might make the optimization process easier.

As noted earlier, it would be somewhat naïve to believe that reality could be replicated by very sophisticated analytical models capable of incorporating all the objectives and constraints of PD management. However, the use of analytical models can be very useful in at least two aspects. First, they avoid the risk of relying exclusively on intuition (science has long demonstrated how intuition can be misleading). Second, the process of developing and discussing models can be an efficient capacity-building process since relevant concepts and trade-offs are part of the debate. Instead of being considered as conflicting, the analytical modeling and subjective expertise of debt managers should be seen as complementary.

Besides being an important strategic planning tool, the benchmark can also provide risk managers with a way to measure performance, comparing the current portfolio with the “optimal” (desirable) one. If they differ greatly, intermediate benchmark portfolios can be chosen for comparison purposes. It is important to note that day-to-day financing strategies-- typically designed by the front-office for short-term tactical actions—should be pursued in a climate with some degree of flexibility, since market conditions may well differ from the scenarios used to devise the medium-term strategy

4.1 Integrated Asset and Liability Management (ALM)

Generally, benchmarks are developed within an Assets and Liabilities Management (ALM) framework. It is perfectly sensible for a government, when assessing its risks, to manage its liabilities in a way that also considers its financial assets. These may vary significantly across countries, but they generally have something in common: The ability or the right to collect taxes. Thus, the characteristics of future primary surpluses become a key-factor in determining optimal debt structure, i.e., the benchmark.

In fact, there is no consensus in the literature on how to conduct an ALM in PD risk management, which is linked to the significant differences in the balances of each government. However, the argument - that it makes no sense to manage liabilities without taking into account the assets of the entity - is sound. Thus, each risk analysis discussed above as well as the development of the benchmark could be based on a portfolio of assets and liabilities. There are some important debates, which go beyond the scope of this chapter, about which sovereign assets should be included in such an analysis. Further, the scope of the analysis is also controversial. As a result, issues such as whether or not to include the monetary and natural reserve bases, for example, and the most relevant level - central government, general government or public among others, remain open for discussion. There seems to be no single answer to apply to all countries, with their widely diverse conditions. More likely, each country will need to customize the ALM approach.

For some years, the National Treasury’s risk and public debt management has applied the ALM analysis. The government’s main financial assets are taken into account in the development, follow-up and projection of risk indicators based on government outstanding debts and financial flows. Similarly, the benchmark analytical model now used considers the government’s main financial assets and their characteristics in the simulation of different portfolios and their implications in terms of cost and risk

4.2 The risk manager and the strategy planning design

As noted in the previous chapter, another important responsibility of the risk manager is to participate in strategic debt planning, which involves designing, monitoring and analyzing the trade-offs among different refinancing strategies that can be implemented by the DMO (in which the front- and middle-offices are also both involved in their design).

One of these responsibilities is to identify possible risks in implementing the debt strategy and define desirable targets for indicators such as outstanding debt, average maturity and others. Usually, these targets are set for the end of the year³⁴ (short-term planning) and for future years (long-term planning). Another responsibility is to monitor the strategy's implementation in order to validate it and, where necessary, propose corrective measures. Occasionally, inconsistencies arise between planning and implementing strategies, mainly due to significant but unforeseeable variations in market conditions; modifications in scenarios usually affect costs and risks of different potential strategies, and may render the original strategy sub-optimal.

Earlier in this chapter, ways were presented to determine long-term public debt objectives (benchmarks). What follows here is a summary of ways to achieve them,³⁵ addressing the transitional debt strategy design and its monitoring.

As noted earlier, such a strategy should consider not only long-term objectives, but also short-term constraints. Silva (2005) divides the process of design, implementation and monitoring into eight stages.

1. Definition of long-term objectives and guidelines;
2. Development of macroeconomic scenarios;
3. Preliminary discussions about scenarios and constraints;
4. Transitional strategy design and preliminary risk assessment;
5. Definition of targets: expected results;
6. Analysis of opportunities and challenges in future years;
7. Tactical debt planning and execution (short term);
8. Monitoring/implementation of the transitional strategy (Annual Borrowing Plan - ABP).

Although risk managers participate in all eight stages, their role is especially important in three: (a) defining the long-term objectives (benchmark), (b) designing the transitional strategy (including the definition of targets for debt indicators), and (c) monitoring the strategy implementation. All these activities were discussed in the previous chapter, but will be detailed further, here

5 Peripheral functions of a public debt risk manager

This section discusses the important role that PD risk managers play in providing more accurate debt dynamics and sustainability exercises. More specifically, it illustrates through a simple example how their risk

³⁴ The National Treasury publishes its Annual Borrowing Plan with the current debt composition and desirable targets for the end of the year.

³⁵ Here, the content of this section is limited to the discussion on strategic planning explored in the previous chapter. For more details see Silva (2005) and Baghdassarian (2003).

analysis and privileged access to information on the debt refinancing strategy can add value to commonly developed debt sustainability analysis.

Debt sustainability has long been a topic of great relevance to policy makers, investors and scholars. Although the main variables that impact the debt trajectory are well known, conventional debt sustainability analyses, typically based on deterministic forecasts, have been limited in scope: Mainly, they have failed to incorporate uncertainty into the model, leading to expected debt indicators that lack a measure of potential dispersion (error).

In recent years, efforts have been made to develop more sophisticated modeling techniques.³⁶ Partly, this is due to the concern about risks associated with macroeconomic shocks and to the growing use of sustainability analysis by economic policy makers to define fiscal targets to control the level of public indebtedness.

PD risk managers can add value to this debate by improving the models for debt sustainability exercises: The means by which they measure other types of risks (such as the *cost-at-risk*) can, with some adaptations, be used to generate stochastic debt dynamics' trajectories. As a result, average expected debt ratios originated from deterministic scenarios can be complemented with a complete probability distribution of such ratios.

Adding uncertainty to debt sustainability analyses may enhance the conclusions drawn from this type of exercise, but may not be a sufficient condition to ensure more precise forecasts. Such assessments usually cover several time frames (generally 5-10 years); during these periods, debt profiles and structures can change substantially as can their sensitivity to different scenarios and types of macroeconomic shocks.

Thus, it is relevant to use assumptions on the refinancing strategy when conducting such analyses. In this regard, risk managers are in a privileged position: Since they actively participate in designing and monitoring the debt strategy, they can perform sustainability tests, including those that consider refinancing strategies. Indeed, it is even more important to include them in countries which have unstable debt structures (that change significantly) or where a large share of the debt will mature in the short term.

To illustrate the advantages of including uncertainty and the refinancing strategy in debt sustainability analysis, a simple exercise using hypothetical data is applied: Its analytical simplicity provides a way to abstract from the implicit methodological complexities and focus on the potential benefits for policy formulation that the aggregation of these two factors can produce.

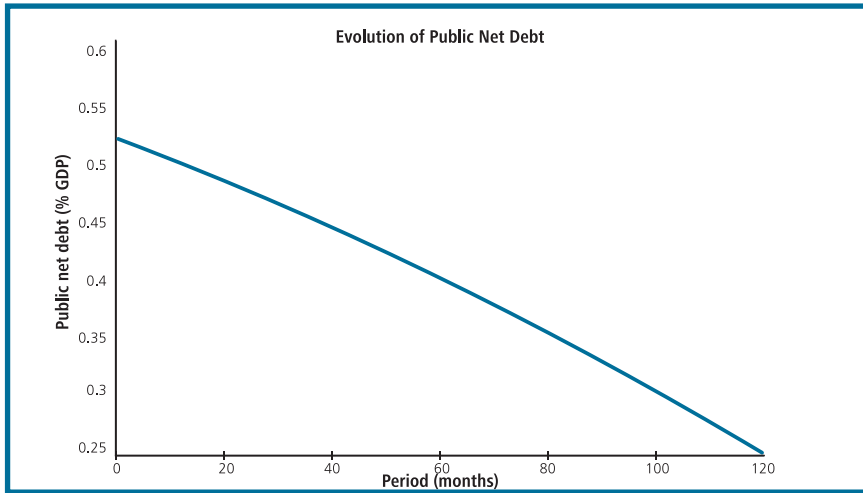
5.1 How to improve debt sustainability analyses - adding uncertainty and the refinancing strategy

The analysis applies basic assumptions for the main determinants of debt dynamics for a 10-year period: nominal interest rates, inflation, GDP growth and primary surplus.³⁷ Starting from a debt/GDP ratio of 51.70% made up entirely of floating-rate instruments, the trajectory of this debt/GDP ratio over the 10 next years can be determined. This deterministic scenario allows for the simplest, though most usual, debt sustainability test. Graph 8 shows the expected trajectory of the debt/GDP ratio.

³⁶ See Barnhill (2003), Xu and Guezzi (2002), Costa, Silva and Baghdassarian (2004).

³⁷ See Annex 2 for details.

Graph 8. Expected trajectory of the debt/GDP ratio



This analysis is then supplemented by the generation of stochastic trajectories for all variables, generating a distribution of debt/GDP ratios across the different horizons foreseen in the tests.

Graph 9. Distribution of debt/GDP ratios across different horizons

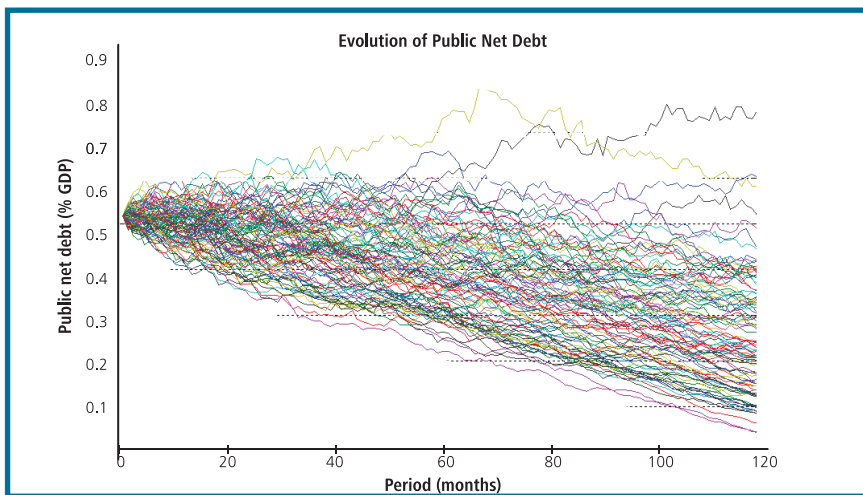


Table 2. Deterministic vs. stochastic simulation results (100% floating rate debt)

Period	Average DL (determ.)	Average DL (stoch.)	Volat	Relat. Volat
0	51.70%	51.70%	0.00%	0.00%
1	49.94%	49.93%	4.25%	8.50%
2	48.19%	48.09%	6.10%	12.69%
3	46.04%	46.00%	7.68%	16.69%
4	43.87%	43.78%	8.86%	20.25%

Period	Average DL* (determ.)	Average DL* (stoch.)	Volat	Relat. Volat
5	41.53%	41.46%	9.90%	23.89%
6	39.17%	39.16%	11.03%	28.18%
7	36.91%	36.85%	12.21%	33.12%
8	34.35%	34.26%	13.24%	38.65%
9	31.68%	31.65%	14.23%	44.97%
10	28.68%	28.39%	14.74%	51.91%

* DL = Debt/GDP.

It should be noted that the models can be calibrated to reflect the values expected from a basic scenario. Both analyses lead to similar expected debt ratios (average), as shown in Table 1, but the information available to economic policy makers becomes broader with the incorporation of stochastic scenarios. For example, those who set fiscal targets can better understand the potential margin of error these targets entail in terms of debt dynamics. In other words, by setting a primary surplus of 4.25% of GDP based solely on the deterministic scenario, they would expect the debt/GDP ratio to fall to 41.53% in 5 years and to 28.68% in 10 years. But, when applied with the stochastic models, the risk these ratios could deviate from their expected values can be assessed. This exercise informs policy makers, for instance, that there is a 95% probability that the debt/GDP ratio will not exceed 57.86% in 10 years.

As mentioned before, another important dimension in sustainability exercises is including “assumptions” about the debt refinancing strategy; and, having easy access to the actual debt strategy the managers intend to implement is an important comparative advantage. The graphs below illustrate the results of debt sustainability tests using the same scenarios and stochastic simulations from the previous exercises, but including the refinancing strategy focused on increasing the share of long-term fixed-rate debt.

Table 3. Deterministic vs. stochastic simulation results (with the refinancing strategy)

Period	Average DL* (determ.)	Average DL* (stoch.)	Volat	Relat. Volat
0	51.70%	51.70%	0.00%	0.00%
1	50.06%	50.11%	1.49%	2.98%
2	48.18%	48.22%	2.13%	4.42%
3	46.39%	46.43%	2.69%	5.80%
4	44.53%	44.58%	3.14%	7.03%
5	42.66%	42.70%	3.52%	8.23%
6	40.51%	40.55%	3.91%	9.64%
7	38.48%	38.52%	4.33%	11.23%
8	36.36%	36.40%	4.69%	12.89%
9	34.34%	34.37%	5.06%	14.73%
10	32.41%	32.44%	5.51%	16.99%

* DL = Debt/GDP.

Graph 10. Expected trajectory of the debt/GDP ratio including a refinancing strategy towards long-term fixed-rate debt

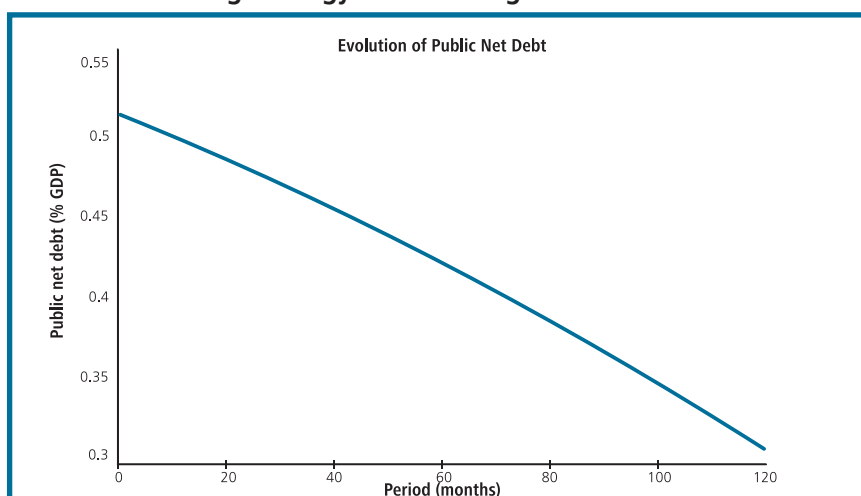


Table 4. Refinancing vs. no-refinancing strategy simulation (deterministic approach)

Period	Without strategy	With strategy	Difference
0	51.70%	51.70%	0.00%
1	49.94%	50.06%	-0.11%
2	48.19%	48.18%	0.02%
3	46.04%	46.39%	-0.35%
4	43.87%	44.53%	-0.66%
5	41.53%	42.66%	-1.13%
6	39.17%	40.51%	-1.34%
7	36.91%	38.48%	-1.58%
8	34.35%	36.36%	-2.01%
9	31.68%	34.34%	-2.66%
10	28.68%	32.41%	-3.72%

In this exercise, floating-rate instruments (with monthly changes in interest rates) comprise 100% of the original debt. The refinancing strategy with 10-year fixed-rate instruments³⁸ yields a higher cost from the interest rate risk premium charged by debt holders in exchange for a lower debt vulnerability to interest rate movements.

The results above reflect the trade-offs in terms of the costs and risks involved in the strategy. By including the refinancing strategy in the analysis, the (average) expected debt/GDP ratio in 10 years increases from 28.68% to 32.41%. Conversely, the distribution of expected debt/GDP ratios is much less dispersed, reflecting reduced exposure to shocks (risk). The introduction of fixed-rate instruments led the relative dispersion (the ratio between one standard deviation and the mean) to fall from 51.91% to 16.99%.

Graph 11. Distribution of debt/GDP ratios across different horizons including refinancing strategy towards long-term fixed-rate debt

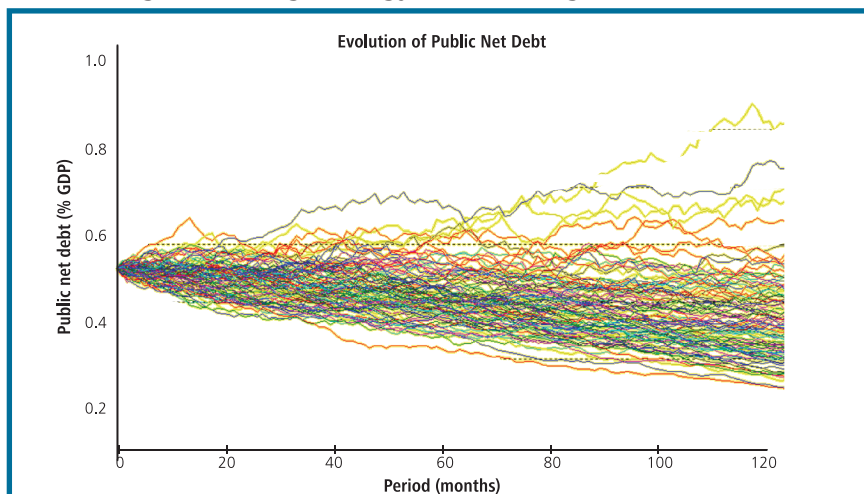


Table 5. Refinancing vs. no-refinancing strategy simulation (stochastic approach)

Period	Without strategy	With strategy	Difference
0	51.70%	51.70%	0.00%
1	49.93%	50.11%	-0.18%
2	48.09%	48.22%	-0.13%
3	46.00%	46.43%	-0.44%
4	43.78%	44.58%	-0.80%
5	41.46%	42.70%	-1.24%
6	39.16%	40.55%	-1.40%
7	36.85%	38.52%	-1.67%
8	34.26%	36.40%	-2.13%
9	31.65%	34.37%	-2.72%
10	28.39%	32.44%	-4.05%

It could be argued that the use of stochastic models to complement simpler exercises based on deterministic scenarios could lead to conclusions that are harder to understand or interpret and depend too greatly on model calibration; also, that the use of alternative deterministic scenarios could lead to a more intuitive analysis of debt sensitivity to changes in macroeconomic variables.

While the relevance of simpler types of analysis should not be minimized, this exercise sheds light on how PD risk managers can complement and enhance the information available to policy makers. Despite the

³⁸ According to the refinancing strategy, 1% of the floating-rate share falls due and is exchanged monthly into ten-year fixed-rate instruments. At the end of this period, 100% of the debt is in fixed instruments.

relative complexity in the design of risk management models, presenting their results in a user friendly way to decision makers is an easy task that has already become widespread, particularly within the financial sector.

6 Concluding remarks

This section described the main responsibilities and contributions of risk managers; in so doing, it offered an overview rather than covering one topic or another in detail, such as risk modeling techniques.

Obviously, the task of describing all the responsibilities of risk managers is not only ambitious but also subject to gaps and criticism. Indeed, some relevant topics, such as the risk of contingent liabilities or credit, were not covered.³⁹

Despite these limitations, the chapter can serve both as a useful guide to those who seek a greater understanding of PD risk management and to illustrate how it is addressed by the National Treasury. Further, in an environment where DMOs around the world are attempting to modernize their risk management practices, the chapter can serve as a starting point, as it provides an overview of the main activities involved. Finally, it can help decision makers involved with issues related to public debt to better explore the skills and analyses PD risk managers can offer.

³⁹ The contingent claims approach as explored by the IMF, see Gapen & Gray (2005) and Barnhill (2003), is an interesting methodology to be explored.

Bibliography

- BAGHDASSARIAN, William. *Avaliação da sustentabilidade fiscal sob incerteza*. Master's dissertation. 2006.
- _____. *Indicadores estocásticos de risco no processo de planejamento estratégico da dívida pública*. Finanças Públicas: VIII National Treasury Award – 2003. Collection of Monographs. Brasília: University of Brazilia press: National Treasury Secretariat, 2004.
- BAGHDASSARIAN, W.; DA COSTA, C. E.; SILVA, A. C. D. *Assessing three models for the analysis of debt sustainability*. National Treasury, 2004.
- BARRO, Robert J. Are government bonds net wealth? *Journal of Political Monetary Economics*, v. 82, p. 1095-1118, 1974.
- BERGSTRÖM, Pal; HOLMLUND, Anders. *A simulation model framework for government debt analysis*. 2000.
- BOLDER, David J. Towards a more complete debt strategy simulation framework. *Bank of Canada Working Paper*, 2002. p.13.
- BONOMO, M.; COSTA, C.; ROCQUE, E.; SILVA, A. *A new framework for debt management in Brazil*. 2003.
- CABRAL, R. S. V. *Tópicos em finanças e dívida pública: prêmio de risco, expectativas, benchmark e cláusulas de ação coletiva*. PhD Thesis, University of Brasília, 2004.
- CABRAL, R. S. V.; LOPES, M. *Benchmark para a Dívida Pública: duas propostas alternativas*. Paper presented during the V Brazilian Meeting on Finance, Jun., 2005. Brazilian Finance Society.
- DANMARK NATIONAL BANK. *Danish government borrowing and debt 2001*. 2001.
- GAPEN, M. T.; GRAY, D. F.; LIM, C. H.; XIAO, Y. Measuring and analysing sovereign risk with contingent claims. *Working Paper*, April 2005. International Monetary Fund (IMF).
- GRANGER, Rita. *Benchmarking for public debt management: the case of Portugal*. Plenary Session 4. Paper presented during the Second Sovereign Debt Management Forum, World Bank, 1999.
- GUIDELINES FOR PUBLIC DEBT MANAGEMENT. *Accompanying document and selected case studies*. International Monetary Fund and The World Bank, 2003.
- _____. International Monetary Fund and The World Bank, 2001.
- LOPES, M. *Composição ótima para a dívida pública: uma análise macroestrutural*. Master's dissertation, University of Brasília, 2003.
- MEDEIROS, Otavio L.; CABRAL, Rodrigo S. V.; BAGHDASSARIAN, Wiliam; ALMEIDA, Mario Augusto. *Public debt strategic planning and benchmark composition*. Paper presented in Inter-American Development Bank, October 2005.
- MENDOZA, E.; OVIEDO, P. *Public debt sustainability under uncertainty*. Inter-American Development Bank, 2003.
- NARS, K. *Excellence in Debt Management: the strategies of leading international borrowers*. Euromoney Publications, 1997.
- OECD. *OECD Studies in Risk Management*. OECD, 2006.
- SILVA, Anderson C. *Strategic planning of the brazilian public debt: the process to design a debt management strategy*. Paper presented in The World Bank, December 2005.
- XU, D.; GUEZZI, P. *From fundamentals to spreads: a fair spread model for High Yield EM Sovereigns*. Global Markets Research – Deutsche Bank, May 2002

Annex 1. The Gerir system

The Gerir system is an important tool in Brazilian PD management, as it allows managers to compare different strategies and understand their trade-offs. Also, it provides sophisticated tools for debt analysis in a probabilistic environment (CaR, CFaR, BaR and VaR).

It is a very flexible system in terms of simulating strategy (issuances, buy-backs, exchanges, etc.), as well as very powerful with regard to generating management indicators (outstanding debt, average maturity of outstanding debt, average maturity of new issuances, maturity structure, duration, profile, etc.).

The system was first developed in 2001 and completed in 2003, when it became a critical part of the process to design strategy. After this, it was improved in order to be more accessible to end users and expand its functions. The figures below are meant to illustrate its flexibility and use.

Figure 1. Gerir system – opening screen

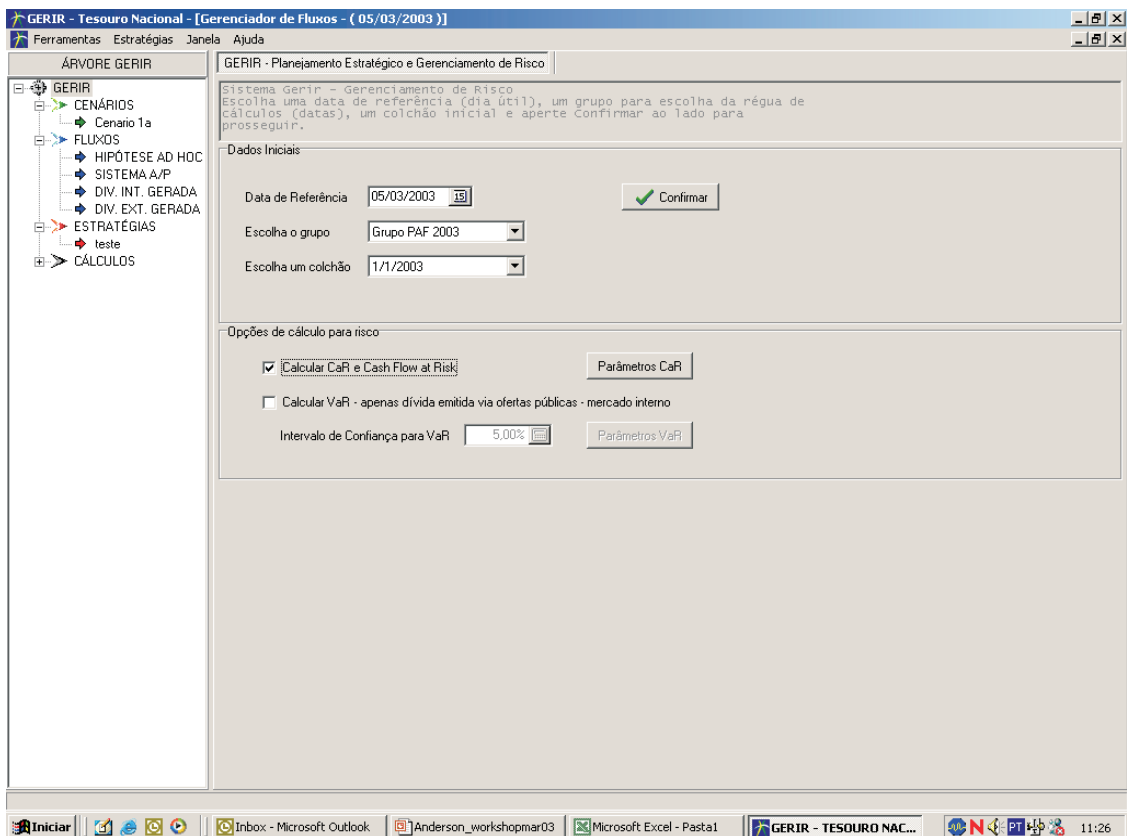


Figure 4. Gerir system – issuance strategies

GERIR - Tesouro Nacional - [Gerenciador de Fluxos - (05/03/2003)]

Ferramentas Estratégias Janela Ajuda

ÁRVORE GERIR

- GERIR
 - CENÁRIOS
 - Cenário 1a
 - FLUXOS
 - HIPÓTESE AD HOC
 - SISTEMA A/P
 - DIV. INT. GERADA
 - DIV. EXT. GERADA
 - ESTRATÉGIAS
 - teste
 - CÁLCULOS

TITULO	DATA OPERAÇÃO	DATA LIQUIDAÇÃO	DATA EMISSÃO	DATA BASE	DATA VENCIMENTO	CUPOM (%)	COMPRA / VENDA	P
NTN-C	01/01/2003	01/01/2003	01/01/2003	01/07/2000	01/03/2006	6,0000	VENDA	
NTN-C	01/01/2003	01/01/2003	01/01/2003	01/07/2000	01/04/2008	6,0000	VENDA	
NTN-C	01/01/2003	01/01/2003	01/01/2003	01/07/2000	01/07/2017	6,0000	VENDA	
NTN-C	01/01/2003	01/01/2003	01/01/2003	01/07/2000	01/04/2021	6,0000	VENDA	
NTN-C	01/01/2003	01/01/2003	01/01/2003	01/07/2000	01/01/2027	6,0000	VENDA	
NTN-P	02/01/2003	02/01/2003	02/01/2003	02/01/2003	02/12/2027	3,2500	VENDA	
LFT	07/01/2003	07/01/2003	07/01/2003	01/07/2000	20/08/2003	0,0000	VENDA	
LFT	07/01/2003	07/01/2003	07/01/2003	01/07/2000	17/12/2003	0,0000	VENDA	
LFT	07/01/2003	07/01/2003	07/01/2003	01/07/2000	19/05/2004	0,0000	VENDA	
LTN	07/01/2003	07/01/2003	07/01/2003	07/01/2003	02/07/2003	0,0000	VENDA	
LTN	07/01/2003	07/01/2003	07/01/2003	07/01/2003	01/10/2003	0,0000	VENDA	
NTN-D	07/01/2003	07/01/2003	07/01/2003	01/07/2000	01/10/2003	12,0000	VENDA	
LFT	14/01/2003	14/01/2003	14/01/2003	01/07/2000	20/08/2003	0,0000	VENDA	
LFT	14/01/2003	14/01/2003	14/01/2003	01/07/2000	17/12/2003	0,0000	VENDA	
LFT	14/01/2003	14/01/2003	14/01/2003	01/07/2000	19/05/2004	0,0000	VENDA	
LTN	14/01/2003	14/01/2003	14/01/2003	14/01/2003	02/07/2003	0,0000	VENDA	
LTN	14/01/2003	14/01/2003	14/01/2003	14/01/2003	01/10/2003	0,0000	VENDA	
NTN-D	14/01/2003	14/01/2003	14/01/2003	01/07/2000	01/10/2003	12,0000	VENDA	
LFT	21/01/2003	21/01/2003	21/01/2003	01/07/2000	20/08/2003	0,0000	VENDA	
LFT	21/01/2003	21/01/2003	21/01/2003	01/07/2000	17/12/2003	0,0000	VENDA	
LFT	21/01/2003	21/01/2003	21/01/2003	01/07/2000	19/05/2004	0,0000	VENDA	
LTN	21/01/2003	21/01/2003	21/01/2003	21/01/2003	02/07/2003	0,0000	VENDA	
LTN	21/01/2003	21/01/2003	21/01/2003	21/01/2003	01/10/2003	0,0000	VENDA	
NTN-D	21/01/2003	21/01/2003	21/01/2003	01/07/2000	01/10/2003	12,0000	VENDA	
LFT	28/01/2003	28/01/2003	28/01/2003	01/07/2000	20/08/2003	0,0000	VENDA	
LFT	28/01/2003	28/01/2003	28/01/2003	01/07/2000	17/12/2003	0,0000	VENDA	
LFT	28/01/2003	28/01/2003	28/01/2003	01/07/2000	19/05/2004	0,0000	VENDA	
LTN	28/01/2003	28/01/2003	28/01/2003	28/01/2003	02/07/2003	0,0000	VENDA	
LTN	28/01/2003	28/01/2003	28/01/2003	28/01/2003	01/10/2003	0,0000	VENDA	

ANALÍTICAS SINTÉTICAS SINTÉTICA-> ANALÍTICA CÁLCULOS CONSOLIDAÇÃO CONSOLIDAÇÃO - COMPLETA DECOI

Seleção Usar a SINTÉTICA Original Consolidar Info. Prim. A/P

Salvar Valor Financeiro Consolidar Parcial

Remover Quantificar Consolidar Estoque

Iniciar [Icons] Inbox - Microsoft O... Anderson_worksho... Microsoft Excel - P... GERIR - TESOUR... [Icons] 11:30

Figure 5. Gerir system – products

Escolha abaixo as telas que deseja visualizar

<input checked="" type="checkbox"/> 01=Maturação na Data do Leilão	<input checked="" type="checkbox"/> 12=Gestão de Ativos e Passivos - Valor Estoque
<input checked="" type="checkbox"/> 02=Estatísticas	<input checked="" type="checkbox"/> 13=Gestão de Ativos e Passivos - Valor Mercado
<input checked="" type="checkbox"/> 03=Gráficos de Maturação	<input checked="" type="checkbox"/> 14=Info. Primárias de A / P - Valor Estoque
<input checked="" type="checkbox"/> 04=Emissão Líquida	<input checked="" type="checkbox"/> 15=Info. Primárias de A / P - Valor Mercado
<input checked="" type="checkbox"/> 05=Maturação no Vencimento	<input checked="" type="checkbox"/> 16=Info. Primárias de A / P - Valor Futuro
<input checked="" type="checkbox"/> 06=Liquidez / Colchão	<input checked="" type="checkbox"/> 17=Evolução do Índice do Colchão
<input checked="" type="checkbox"/> 07=Prazo Médio	<input checked="" type="checkbox"/> 18=Gráficos de Evolução do Índice do Colchão
<input checked="" type="checkbox"/> 08=Gráficos de Prazo Médio	<input checked="" type="checkbox"/> 19=Risco de Taxa de Juros
<input checked="" type="checkbox"/> 09=Gráficos de Perc. a Vencer	<input checked="" type="checkbox"/> 20=Risco Cambial
<input checked="" type="checkbox"/> 10=Gráficos de Composição do Estoque	<input checked="" type="checkbox"/> 21=CaR, Cash Flow e BaR
<input checked="" type="checkbox"/> 11=Diretrizes do PAF	<input checked="" type="checkbox"/> 22=Value at Risk

Figure 6. Gerir system – maturity structure

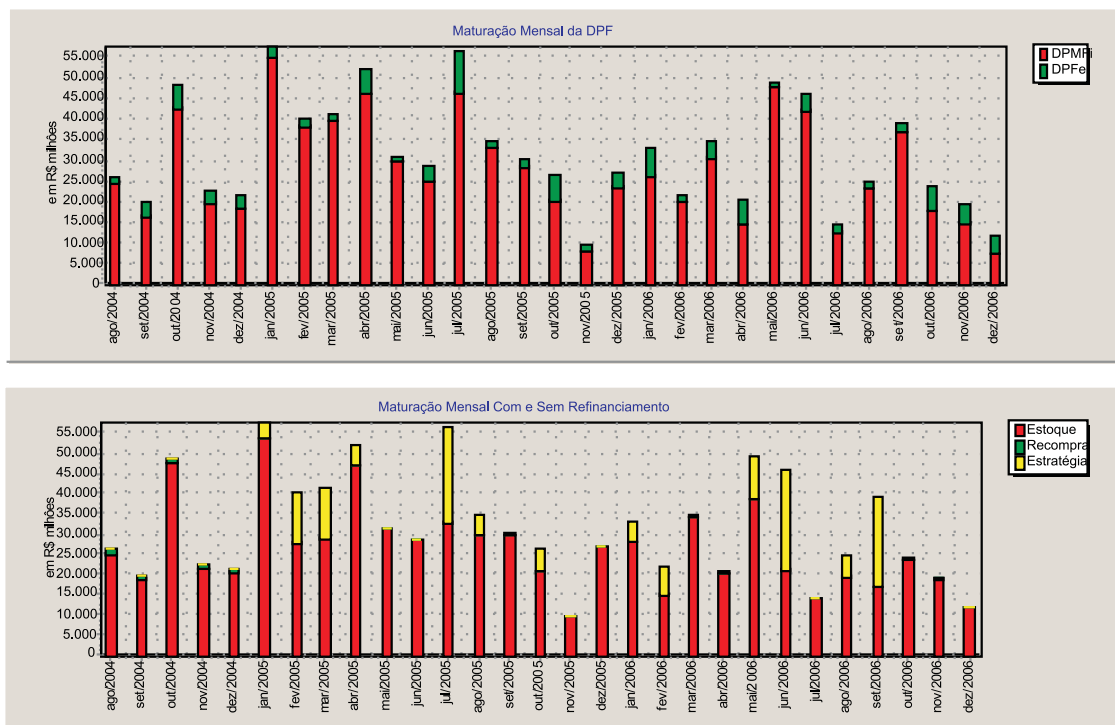


Figure 7. Gerir system – liquidity, average maturity and sensitivity to interest rates

CfaR - DPF - 2003 [PAF básico 11.09.06] : Emissão Líquida				
R\$ milhões		01.11.2006	01.12.2006	01.01.2007
DATA				
Emissão	LFT	0	0	0
	LTN	0	0	0
	NTN-B	0	0	0
	NTN-C	0	0	0
	NTN-D	0	0	0
	NTN-F	0	0	0
	Demais	0	0	0
	Sub-Total int.	0	0	0
	Dívida Externa	0	0	0
	Total	0	0	0
Resgate	LFT	21,115	21,345	21,582
	LTN	1,369	1,386	1,403
	NTN-B	672	681	690
	NTN-C	0	0	0
	NTN-D	10,251	10,385	10,522
	NTN-F	0	0	0
	Demais	51	69	147
	Sub-Total int.	33,457	33,865	34,344
	Dívida Externa	2,362	176	2,880
	Total	35,820	34,041	37,223
Emissão Líquida	LFT	-21,115	-21,345	-21,582
	LTN	-1,369	-1,386	-1,403
	NTN-B	-672	-681	-690
	NTN-C	0	0	0
	NTN-D	-10,251	-10,385	-10,522
	NTN-F	0	0	0
	Demais	-51	-69	-147
	Sub-Total int.	-33,457	-33,865	-34,344
	Dívida Externa	-2,362	-176	-2,880
	Total	-35,820	-34,041	-37,223

CfaR - DPF - 2003 [PAF básico 11.09.06] : Prazo Médio							
	Brades	Globals	Euros	Ienes	Mobiliária	Contratual	Total
nov/2006	38.73	82.13	43.26	0.00	75.98	0.00	75.98
dez/2006	37.73	81.91	47.21	0.00	76.84	0.00	76.84
jan/2007	36.70	81.03	46.16	0.00	75.89	0.00	75.89
jan/2008	32.00	80.39	43.72	0.00	75.28	0.00	75.28

Prazo Médio de Emissão da Dívida Interna / Prazo Médio de Estoque da Dívida Interna

CfaR - DPF - 2003 [PAF básico 11.09.06] : Risco de Taxa de Juros					
Taxa Selic (em p.p.)	14.04				
Varição na Taxa Selic (em p.p.)	1.00				
Em R\$ milhões					
Impacto no estoque de aumento de 1.00 p.p. na taxa selic em outubro 2006					
	estoque inicial	estoque final a 15.04%	estoque final a 14.04%	impacto	impacto acumulado
02/10/2006	65,108.57	--			
02/11/2006		65,873.22	65,825.31	47.91	47.91
02/12/2006		66,646.85	66,549.94	49.00	96.91
02/01/2007		67,429.56	67,282.55	50.11	147.02
02/02/2007		68,221.47	68,023.22	51.24	198.25
02/03/2007		69,022.68	68,772.05	52.38	250.63
02/04/2007		69,833.30	69,529.12	53.55	304.18
2006		67,397.22	67,252.28	144.93	144.93
02/10/2007		74,900.89	74,249.81	651.09	651.09
2007		77,533.76	76,694.50	694.32	694.32
2008		89,229.08	87,493.90	895.93	1,735.18

Crítério de Caixa - Dívida Total / Crítério de Liquidez

Figure 8. Gerir system – stochastic models

GERIR - Tesouro Nacional - [Cadastro dos Parâmetros do CaR]

Ferramentas Estratégias Janela Ajuda

Número de Simulações: 5000 IC do CaR: 95% Correlação

Data de Hoje: 15/04/2002 Intervalos do Hist: 50 Selic/Dolar: -12%

Todas as simulações são baseadas na Data CaR Selic/IGPM: -20%

Data CaR: 02/12/2013 Dolar/IGPM: 40%

Dias Úteis do CaR: 2.927 Construir Curvas

SELIC DOLAR PRE IGPM Orçamento

Parâmetros para construção das curvas SELIC

MODELO CIR/vasicek:
 $dr(t) = b*[a - r(t)]*dt + c*r(t)^(d)*dz(t)$

Estimados com dados diários da taxa SELIC diária

d: 1/2 - Cir

velocidade (b): 0,006882	Empírica	Neutra ao Risco
média (a): 0,0006379	Vida Média (em dias): 100,72	122,49
volatilidade (c): 0,0001492	Média Anual Efetiva: 17,44%	21,59%
Parâmetro de Risco (PR): -0,0012231483705	Volatilidade Anual: 0,24%	

Digite a Selic Inicial ou entre 0 (zero) para pegar do Banco de Dados

Selic Inicial: 0%aa

Windows Taskbar: Iniciar, GERIR - TESOURO NAC..., Microsoft PowerPoint - [A...], 16:40

Annex 2. Public debt simulations: methodology for the empirical exercise presented in section 5 of this chapter

Usually, sustainability exercises consider deterministic macroeconomic scenarios as well as hypotheses about the level of the primary surplus and the seignorage. However, there are at least two other aspects that should be considered to obtain more accurate estimates. The first regards the uncertainty related to the scenarios and the second, the public debt profile.

Although a brief discussion about these topics was presented in Section 3, this annex includes a methodological discussion about the model, which presents the hypothesis and parameters used in Section 5.

First, four simulations are produced. The first can be considered a benchmark scenario since it involves no uncertainty or refinancing strategy. The second exercise includes uncertainty, although no refinancing strategy. The third uses a deterministic scenario to assess the impact of a refinancing strategy. Finally, the fourth considers not only the refinancing strategy but uncertainty, as well.

It is important to present the general framework applied to include uncertainty in the simulations; this is the Monte Carlo simulation, which generates thousands of macroeconomic scenarios. Also, the CIR (Cox-Ingersoll-Ross) model was used to generate interest rate scenarios and the geometric Brownian motion⁴⁰ was applied for GDP and inflation.

Besides these models, the traditional Blanchard model is used to simulate debt evolution, as shown by the equation below:

$$d_t = \left(\frac{1+r}{1+n} \right) d_{t-1} - (t_t - g_t) - \frac{M_t - M_{t-1}}{p_t * PIB} \quad (9)$$

where:

d_t	– Net debt in t as a proportion of GDP
r	– Real interest rate
n	– Real GDP growth rate
t_t	– Taxes as a proportion of GDP
g_t	– Expenditures as a proportion of GDP
M_t	– Monetary base in t
p_t	– Current price level
PIB	– Gross domestic product

All models have been calibrated to reflect long-term expectations rather than current levels. If the latter had been adopted, other peripheral discussions would probably have emerged.

⁴⁰ See Baghdassarian (2006) for more details about these models.

The first exercise considers a deterministic approach to generate scenarios and maintains 100% of the debt profile in floating rate securities. This is based on a primary surplus of 4.25% of GDP, annual inflation about 3%, nominal interest rates around 11% a year, and initial net debt of 51.7% of GDP. From these parameters, the evolution of the net debt for the next 10 years was calculated. An important hypothesis is that all floating rate debt falling due is also refinanced with the same instrument.

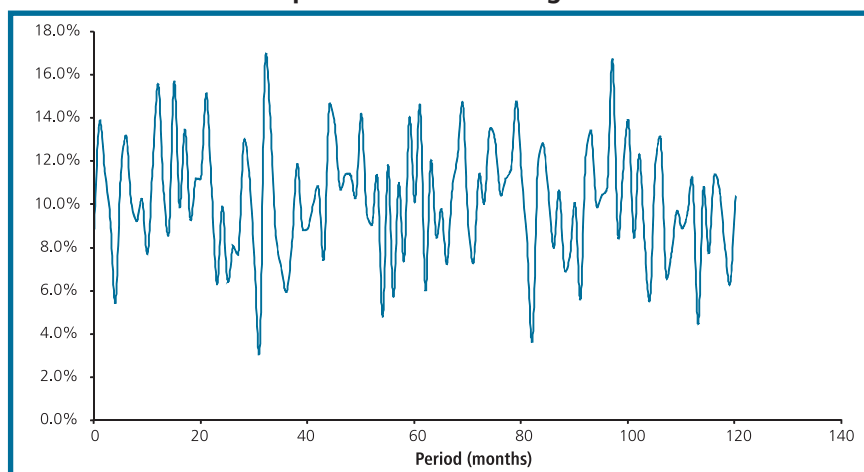
The second exercise is similar to the first, but instead of a deterministic scenario, a thousand different scenarios are used to analyze uncertainty around the conclusions of the first exercise. Following table presents the evolution of the net debt, both in the determinist approach and in the stochastic cases. In the latter, the mean and the standard deviation are used to express the results.

Table 6. Deterministic versus stochastic scenarios generation (without refinancing strategy)

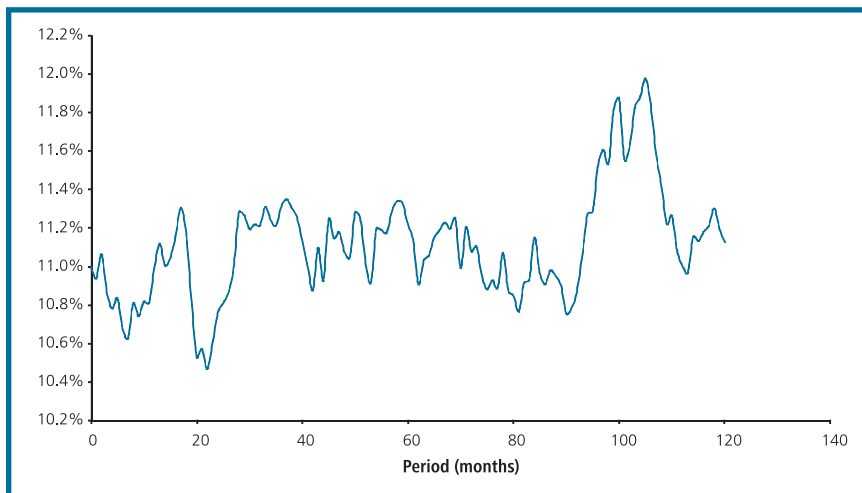
Period	Average DL (determ.)	Average DL (stoch.)	Volat	Relat. Volat
0	51.70%	51.70%	0.00%	0.00%
1	49.94%	49.93%	4.25%	8.50%
2	48.19%	48.09%	6.10%	12.69%
3	46.04%	46.00%	7.68%	16.69%
4	43.87%	43.78%	8.86%	20.25%
5	41.53%	41.46%	9.90%	23.89%
6	39.17%	39.16%	11.03%	28.18%
7	36.91%	36.85%	12.21%	33.12%
8	34.35%	34.26%	13.24%	38.65%
9	31.68%	31.65%	14.23%	44.97%
10	28.68%	28.39%	14.74%	51.91%

The following graphs show the monthly evolution of nominal GDP growth and nominal interest rates in the stochastic environment (in mean terms).

Graph 12. Nominal GDP growth



Graph 13. Nominal interest rate evolution



In the third and the fourth exercises, instead of using a fixed portfolio (100% floating debt), the effect of refinancing on net debt evolution is considered. Again, results can be evaluated using either a deterministic or a stochastic approach.

Here, the same macroeconomic hypothesis is used as in the first exercise, but it adopts a strategy of change in the debt profile, with an increase in the fixed share of 1% a month. Table 7 shows the evolution of the net debt in this simulation.

Table 7. Generation of deterministic vs. stochastic scenarios (with a refinancing strategy)

Period	Average DL (determ.)	Average DL (stoch.)	Volat	Relat. Volat
0	51.70%	51.70%	0.00%	0.00%
1	50.06%	50.11%	1.49%	2.98%
2	48.18%	48.22%	2.13%	4.42%
3	46.39%	46.43%	2.69%	5.80%
4	44.53%	44.58%	3.14%	7.03%
5	42.66%	42.70%	3.52%	8.23%
6	40.51%	40.55%	3.91%	9.64%
7	38.48%	38.52%	4.33%	11.23%
8	36.36%	36.40%	4.69%	12.89%
9	34.34%	34.37%	5.06%	14.73%
10	34.34%	32.44%	5.51%	16.99%

As expected, the floating rate strategy is less costly than that of the fixed-rate. If it includes two standard deviations, while the fixed-rate strategy could take the debt to values around 43.5% of GDP, the floating-rate strategy could take them to 57.9%. Therefore, depending of the degree of risk aversion, 2.8% is not expensive because it protects the debt against a potential increase of nearly 14% of GDP.

Annex 3. The long-term benchmark

In recent years, the National Treasury has worked on an ALM analytical framework in order to analyze and identify the desired public debt profile in the long-term (the benchmark). As mentioned before, this topic has received growing attention from governments, multilateral organizations and scholars. However, there is no consensus in the literature regarding the appropriate methodology.

The National Treasury recently developed a state-of-the-art risk management framework as well as analytical models that help define a benchmark for public debt. These models become important tools for decision-makers as they allow them to determine the debt structure the government wants, based on its cost and risk preferences.

In summary, the benchmark is an optimal long-term structure used to guide short- and medium-term financing strategies, and is an important risk management and strategic planning tool. In a steady state, it illustrates the trade-off between cost and risk for efficient debt profiles. Based on the information generated by the model, debt managers can also evaluate the performance of financing operations, including both new issuances and those exclusively aimed at risk management.

1 Methodological notes

The most popular indicator among analysts and investors is the Public Sector Net Debt-to-GDP (PSND/GDP) ratio because of its broader scope when compared to the Federal Public Debt, which only includes Central Government liabilities. For that reason, the Brazilian benchmark model considers the PSND/GDP ratio a relevant debt measure, even if its scope is bigger than the debt for which the Government is responsible. In an inter-temporal analysis of the Government's budget constraints, all of its assets and liabilities should be considered when evaluating the country's fiscal situation. The model is therefore inserted in an ALM context.⁴¹

Finally, the benchmark model assumes the economy has already reached a steady state, which means that all relevant variables are at their long-term equilibrium values. This seems to be consistent with the search for an optimal long-term profile. The stationary scenario has the following characteristics: a stable economic environment reduced fiscal vulnerability, sovereign bonds with a credit rating compatible to the investment grade rating, domestic interest rates compatible with international levels, inflation under control, and sustainable economic growth. This scenario is expected to be fully attained in the following years, so the analysis would move from this point onwards

2 The stochastic model

2.1 Scenarios

The cost of carrying the debt is determined by the evolution of its indicators, i.e., the different interest, exchange and inflation rates. In the stochastic financial approach, each relevant variable is determined by specific stochastic processes (described next).

⁴¹ In fact, most economic analysts and market participants, as well as international organizations and rating agencies, consider the PSND/GDP ratio as the most relevant debt sustainability indicator.

A CIR⁴² model is used for the short-term interest rate (Selic) and belongs to the class of one-factor equilibrium models. In other words, the interest rate process (risk neutral) can be described as:

$$dJ_t = \alpha (J^* - J_t)dt + \sigma_1 \sqrt{J_t} dz_t^1 \quad (1)$$

Where:

- J_t – Interest rate (Selic) at t
- α – Parameter of speed of return to the average
- J^* – Average long-term interest rate
- σ_1 – Interest rate volatility
- dz_t^1 – Wiener process

The price index follows a geometric Brownian motion:

$$dI_t = \mu I_t dt + \sigma_3 I_t dz_t^3 \quad (2)$$

Where:

- I_t – Price index at t
- μ – Average price index growth rate
- σ_3 – Price index volatility
- dz_t^3 – Wiener process

For the real exchange rate, a CKLS⁴³ model is used with the exchange rate exponent in the volatility term equal to one ($\gamma=1$). This process is described as follows:

$$dC_t = \beta (C^* - C_t)dt + \sigma_2 C_t^\gamma dz_t^2 \quad (3)$$

Where:

- C_t – real exchange rate in t
- β – speed of return to the average
- C_t^γ – long term average of the real exchange rate
- σ_2 – real exchange rate volatility
- dz_t^2 – Wiener process

⁴² Cox, Ingersoll and Ross model. See Hull (1998).

⁴³ The CKLS model (Chan, Karolyi, Longstaff and Sanders) is a generalization of the CIR model.

However, the carrying cost of the exchange debt depends not on the real rate but rather on the nominal exchange rate. Nonetheless, the nominal rate of the real rate can be obtained if domestic and external price indices are available. The former is available, while for the latter, the following deterministic process is applied:

$$dI_t^e = \mu^e I_t^e dt \tag{4}$$

Where:

- I_t^e – External price index at t
- μ – Interest rate (Selic) at t

The nominal exchange rate can be calculated, by definition, as:

$$N_t = \frac{I_t}{I_t^e} C_t \tag{5}$$

Using Ito’s Lemma in this last equation and the real exchange rate diffusion process as well as the domestic and external price indices, the nominal exchange rate is computed. Each of the three primitive processes (for interest rates, real exchange rates and inflation) has a stochastic term characterized by a Wiener process. However, in practice, these variables are correlated.

Economic relations among these variables would make it difficult, for example, to imagine a situation where all have their values simultaneously increased over time. As a result, with the aim of adding some macroeconomic consistency to the model, it is reasonable to postulate some correlation structure among them, for which the Cholesky decomposition model is used to create random (pseudo) correlated numbers.

2.2 Security prices

The debt-carrying cost depends on the issuance cost of each security. For example, according to their remuneration characteristics, four types of instruments currently used by the National Treasury include: (a) the fixed issuance rate, for LTNs and NTN-Fs; (b) the interest coupon added to inflation, for the NTN-Bs; (c) the Selic rate, for LFTs (floating rate); and (d) the interest coupon added to the exchange rate variation for external securities.

LFTs pay exactly the compounded overnight interest rate in the security period; and, it can be assumed it is always sold at par, i.e., at a price equal to its face value. Therefore, its ex-post cost will be the compounded interest rate in the period.

Since LTNs/NTN-Fs are fixed securities, their carrying cost will obviously be the rate at which the securities were issued. The price of the fixed security is calculated according to the CIR analytical solution and uses the formula of equation 6 below:

$$P(t,T) = A(t,T)e^{-B(t,T)J} \tag{6}$$

Where:

$$B(t, T) = \frac{2(e^{\gamma(T-t)} - 1)}{(\gamma + a + \lambda)(e^{\gamma(T-t)} - 1) + 2\gamma}$$

$$A(t, T) = \left[\frac{2\gamma e^{(a+\gamma+\lambda)(T-t)/2}}{(\gamma + a + \lambda)(e^{\gamma(T-t)} - 1) + 2\gamma} \right]^{2\alpha J^* \cdot 1 / \sigma_1^2}$$

$$\gamma = \sqrt{(a + \lambda)^2 + 2\sigma^2}$$

$$R(t, T) = -\frac{1}{T-t} \ln A(t, T) + \frac{1}{T-t} B(t, T)r(t)$$

λ is known as the risk premium parameter and its function in this case is to adjust the interest curve in the model to that of the steady state. In theory, this parameter is essential for risk-neutral pricing.

The prices of inflation-linked and exchange rate linked securities depend on fixed-rate securities of equivalent maturity. The coupon of the domestic issuance of an inflation-linked security ($C_{\text{inflation}}$) corresponds to the rate of a fixed security of equivalent maturity, adjusted to discount the expected inflation, as shown in the ratio below:

$$C_{\text{inflation}}(t, T) = \frac{R(t, T)}{(1 + \mu)} \quad (7)$$

In a similar way, the coupon of a Foreign Exchange (FX)-rate-linked security (C_{FX}) corresponds to the rate of a security of similar maturity adjusted by the expected exchange rate variation (Δ):

$$C_{\text{FX}}(t, T) = \frac{R(t, T)}{(1 + \Delta)} \quad (8)$$

Aside from the expected inflation or the expected exchange rate variation, the price of these securities is also adjusted by a risk premium. This premium represents how much lower the rate of an inflation-linked or FX-linked security should be, relative to a fixed security of equivalent maturity. In other words, each premium represents the reduction applied to the rate fixed in *reais* to obtain the fixed rate of an external issuance or, alternatively, the coupon of an inflation-linked issuance, excluding the expected inflation and exchange rate variations.

These premiums are included in the model using the Nelson-Siegel procedure, which associates a premium (P) to the term (T) in view of parameters b_0 , b_1 , b_2 and k , as seen in the formula below:

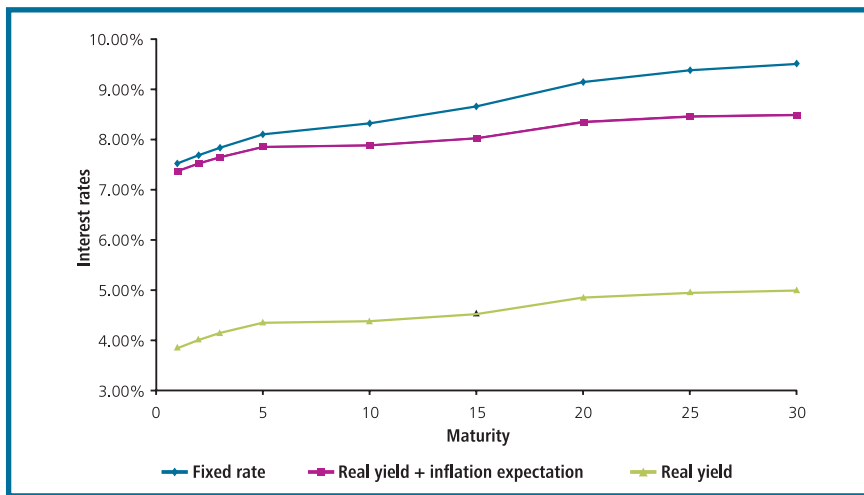
$$P = \beta_0 + (\beta_1 + \beta_2 \cdot T) \cdot e^{-kT} \quad (9)$$

With inflation-linked securities, it makes sense to consider that fixed-rate securities present a risk premium to compensate investors for the uncertainty regarding inflation. For external securities, it is also reasonable

to conceive a theoretical positive premium of real-fixed securities vis-à-vis those fixed in dollars or euros, as investors will be protected against the volatility of fluctuations in the price in *reais*.

The use of the Nelson-Siegel procedure is better understood when analyzing the external interest curves resulting from the fixed domestic curve minus the Nelson-Siegel premiums and the expected FX-rate variations. Likewise, the inflation coupon curve is computed as the fixed curve minus the expected inflation and the Nelson-Siegel premium.

Graph 14. Nelson-Siegel premiums



Thus the model generates the dynamics for the main macroeconomic variables (interest, inflation and exchange rates), as well as the issuance price of each type of security.

Now, it is possible to derive the debt dynamics and calculate the carrying cost for any given debt.

2.3 Debt-carrying costs

As the hypothesis assumes that LFTs are sold at par, their carrying cost is simply the Selic rate in the period:

$$R_t^{LFT} = J_t \tag{10}$$

The carrying cost of LTNs/NTN-Fs in each period is a weighted average of the issuance costs of all fixed securities in the outstanding debt. It is computed as:

$$R_t^{LTN} = \sum_{s=0}^n \omega_{t-s} r_{t-s} \tag{11}$$

where ω_{t-s} is the percentage in t of fixed debt issued in $(t-s)$ and r_{t-s} is the issuance cost of the fixed security in $(t-s)$.⁴⁴

For FX-linked securities, the carrying cost is formed by the evolution of the nominal exchange rate and the weighted rate of the coupon of those securities in the outstanding debt. This coupon average R_t^C is calculated in a similar way:

$$R_t^C = \sum_{s=0}^n \omega_{t-s}^c r_{t-s}^c \quad (12)$$

where ω_{t-s} is the percentage in t of the FX-rate debt issued in $(t-s)$ and r_{t-s}^c is the rate of the security coupon issued in $(t-s)$.

As a result, the carrying cost of FX-rate-linked securities is given by:

$$R_t^{FX} = \left(1 + \frac{dN_t}{N_t}\right) (1 + R_t^C) - 1 \quad (13)$$

The case of inflation-linked securities (NTN-B) is very similar to that of FX-rate-linked securities. The interest coupon in each period, R_t^I , is also calculated as a weighted average of the rates of outstanding coupons

$$R_t^I = \sum_{s=0}^n \omega_{t-s}^i r_{t-s}^i \quad (14)$$

where ω_{t-s}^i represents the percentage in t of the inflation-linked debt issued in $(t-s)$ and r_{t-s}^i is the rate of the security coupon issued in $(t-s)$.

The carrying cost of NTN-Bs is given by:

$$R_t^{NTN-B} = \left(1 + \frac{dI_t}{I_t}\right) (1 + R_t^I) - 1 \quad (15)$$

Therefore, for a given debt profile, its carrying cost is determined by the weighted average of the carrying cost of each type of security, as previously derived. In other words:

$$R_t^D = \lambda_{LFT} R^{LFT} + \lambda_{LTN} R^{LTN} + \lambda_{FX} R^{FX} + \lambda_{NTN-B} R^{NTN-B} \quad (16)$$

where R_t^D is the carrying cost of the debt portfolio and λ_{LFT} , λ_{LTN} , λ_{FX} , λ_{NTN-B} represent the share of each type of security in the given debt profile.⁴⁵

⁴⁴ Each month, 1/12 of the initial outstanding volume of fixed securities matures and is replaced by a new issuance. As a result, the typical weight of the issuance of a one-year fixed security is 1/12. For a five-year security, the weight will be 1/60.

⁴⁵ Obviously, the sum of the shares should total one.

3 Debt dynamics

Given the debt-carrying cost expressed by equation (16) - R_t^D , the current outstanding Federal Public debt (FPD), the monetary base variation (ΔM_t) and the primary surplus (ps_t), it is possible to derive an equation for the public debt dynamics:

$$FPD_{t+1} = (FPD_t - \Delta M_t - ps_t)(1 + R_t^D) \quad (17)$$

However, although the National Treasury has direct control only over the Federal Public Debt, the relevant sustainability indicator to be monitored is the PSND/GDP ratio.

When equation (17) is manipulated, equation (18) is obtained.

$$NPSD_t = FPD_t + M_t + SelicLiab_t + USDLiab_t - Reserves_t - GDPAssets_t - SelicAssets_t + Other_t \quad (18)$$

The different assets and liabilities that make up the PSND can be groups in the following categories: FPD, monetary base (M), floating rate public sector liabilities (SelicLiab), dollar-indexed public sector liabilities (USDLiab), international reserves (*Reserves*), public sector assets correlated to GDP (GDP Assets), floating rate public sector assets (SelicAssets), and other assets and liabilities.

GDP follows a geometric Brownian motion similar to the domestic price index process (equation 2). The monetary base increases at the same rate as nominal GDP, as in a quantitative theory framework. The evolution of reserves depends on the Central Bank's purchase/sale projections. Other variables depend on the evolution of the dollar, the Selic rate or GDP; these represent residual liabilities that depend on other forms of indexation.

4 How to use the model

The general idea of the model is relatively simple, as seen in Figure 6. Monte Carlo simulations are used to derive an efficient cost and risk frontier for the public debt. As usual, a debt profile is efficient when its associated cost is the lowest possible in view of the risk chosen. The set of all efficient profiles defines the efficient frontier and reflects the trade-off between cost and risk that debt managers face.

Several stochastic scenarios are generated to describe how the main macroeconomic variables (interest, exchange and inflation rates) and the price of securities evolve over time. Although stochastic (pseudo), as mentioned before, the model equations are correlated so as to ensure macroeconomic consistency.

Once many trajectories for the main macroeconomic variables in the selected period are achieved, a given debt portfolio is fixed, based on a debt composition representing the FPD, which differs in terms of its return and maturity characteristics. Assuming this portfolio is kept constant over time,⁴⁶ different stochastic trajectories are simulated for the Public Sector Net Debt-to-GDP ratio in a steady state

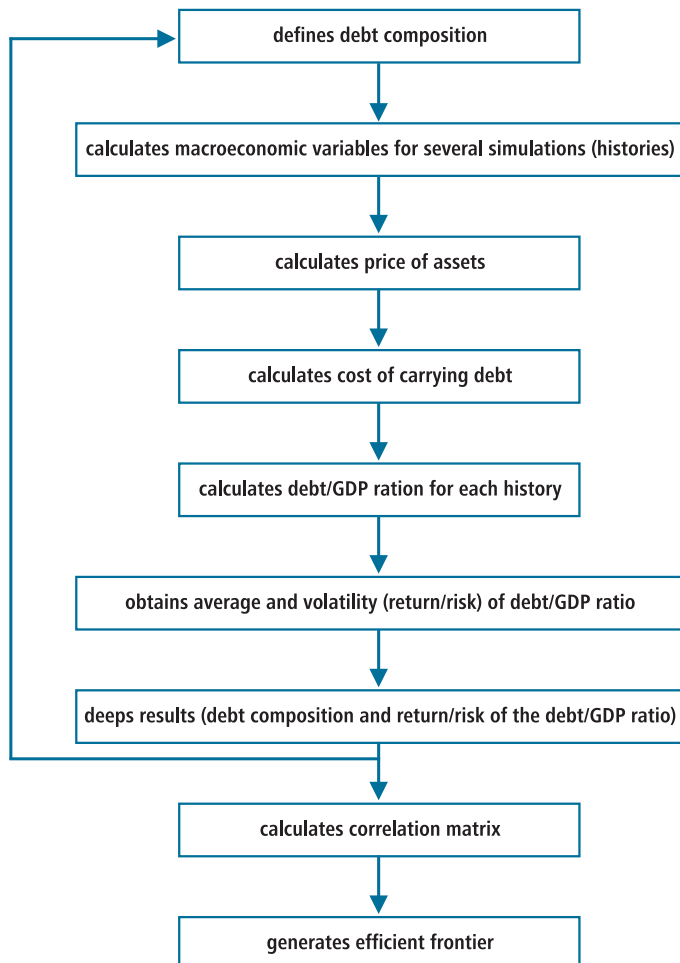
In Brazil, a debt portfolio can include four types of instruments, according to their return characteristics: fixed securities, floating rate securities (linked to the overnight rate), inflation-linked securities, and FX-rate-linked securities. Each instrument can have different maturities in order to create a set of representative short-, medium- and long-term securities. In the model, the debt instruments considered are:

⁴⁶ This objective is achieved by a maturing structure that is constant over time.

- Fixed securities (1, 2, 5 and 10 years);
- Floating rate securities (5 years);
- Inflation-linked securities (10, 20 and 30 years);
- FX-rate-indexed securities (dollar: 10 and 30 years; euro: 15 years).

Each simulated trajectory generates different carrying prices and costs for each security. As a result, a different carrying cost is derived for each trajectory. Thus, a large number of PSND/GDP ratios can be computed to obtain a probability distribution associated with each possible debt profile. The analysis focuses on this distribution at the end of the simulation period (10 years), from which the cost and risk measures are taken.⁴⁷

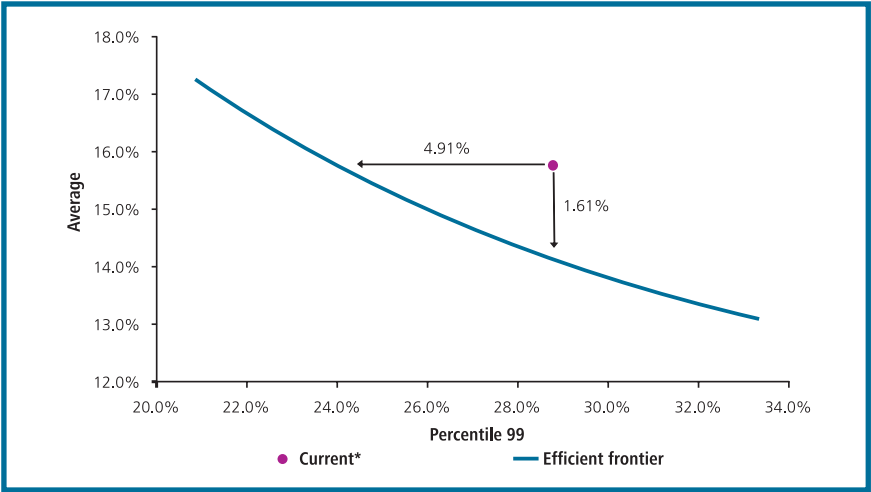
Figure 9. Schematic summary of the model dynamics



⁴⁷ The average of the PSND/GDP ratio is considered to be the cost indicator and the 99th percentile as the risk measure. It is important to highlight that FPD profiles are used in order to generate cost and risk measures of the PSND/GDP ratio. As mentioned before, this is justified by the fact that this last indicator is the one most seen as a proxy for sustainability, although the Treasury only has control over the FPD.

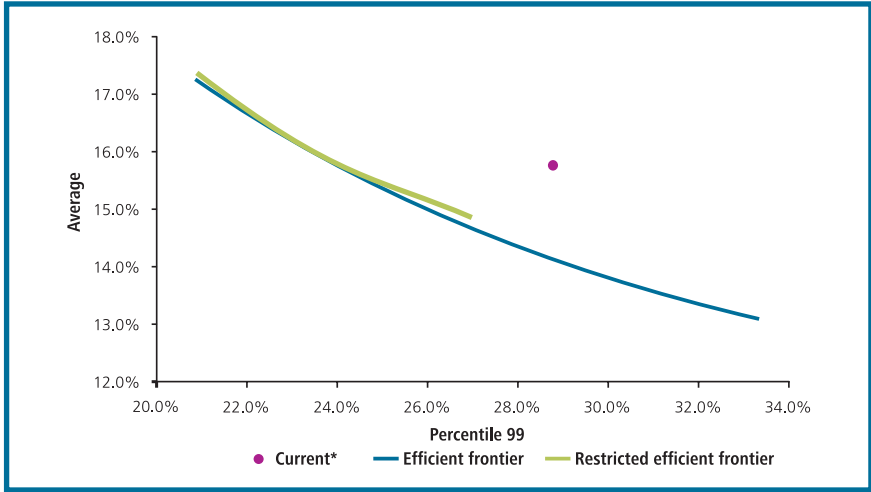
The dot above the curve on the right indicates the current PSND/GDP position in terms of cost and risk, in view of the current FDP profile. The model therefore suggests that, under steady state conditions, it would be possible to take 1.61% off the costs by keeping the risk level constant or reduce the risk by 4.91% by keeping the cost unchanged.

Graph 15. The efficient cost and risk frontier



Finally, the Government’s behavior towards risk allows for choosing an optimal specific profile of the efficient frontier known as the benchmark. The efficient frontier allows us not only to compare the situation of any profile relative to the frontier, but also provides decision-makers with a complete menu of possible efficient choices. Evidently, the choice of a particular portfolio as the public debt benchmark implies choosing the risk level the Government (and consequently society) is willing to bear. In view of a desired risk level, the corresponding portfolio can be taken from the frontier.

Graph 16. The efficient cost and risk frontier under profile constraints



* Under steady state conditions

The model is interesting because it also allows for imposing constraints on efficient profiles, as seen in Figure 8. This adds it considerable flexibility, since there are important risk dimensions that are not directly captured by the frontier but which can be included as constraints: For example, it is possible to introduce constraints as regards the average maturity of the maximum debt percentage falling due in 12 months.

Graph 16 shows the frontier resulting from the addition of the following constraints: maximum of 20% of external debt (exchange), 30% LFT ceiling, and maximum of 30% of debt maturing in 12 months.

5 Concluding remarks

In line with best international practices, the National Treasury develops and submits the results of benchmark (efficient portfolio) studies to the fiscal policy manager (the Minister of Finance or Executive Committee), who is responsible for choosing the acceptable risk level for the Government. Topics related to debt sustainability are also considered, when defining maximum acceptable cost levels.

As mentioned before, although good management of the trade-off between cost and risk suggests the use of traditional financial tools, there are some factors peculiar to governments that impede the indiscriminate use of the financial theory in public debt analyses. Like most countries, Brazil's stated debt management objective is to minimize long-term costs and keep prudent risk levels. Other objectives involve developing the secondary market, expanding the investor base, and building a yield curve, which is a benchmark for pricing public and private assets.

Further, it is worth mentioning that the National Treasury has developed an alternative benchmark model as well, now at an initial stage, which is also based on efficient portfolios, using the framework of a macro-structural model to describe the evolution of the economy. This model could complement the one described here. Recently, the National Treasury improved the two approaches. A special challenge lies in achieving a good specification of a macro-model, particularly for emerging economies. However, the use of different analytical models could translate into important gains in terms of complementarity and understanding of the theme.

Annex 4. Methodological differences between average life and average maturity

There is a consensus in debt management in Brazil that the correct way to calculate the average maturity—to reflect as accurately as possible the refinancing risk—is to use a formula similar to the Macaulay duration and the refixing duration. However, a few years ago, the National Treasury had to incorporate the concept of average life so as to make Brazilian debt internationally comparable.

Equation (19) expresses the methodology the Treasury applied to calculate the average maturity of the public debt. The most important differences are the interest rate used to discount cash flows (Macaulay duration) and the weight factor - T_i (refixing duration).

$$AL = \frac{1}{PV} \times \sum_{i=1}^n PV_i \times T_i \quad (19)$$

For the National Treasury, the original security issuance rates are used to discount their cash flows; and, the variable T_i always measures the time lag between now and each of the cash flows (coupons and principal).

The average life, in turn, as shown below, considers only principal payments for each security. As a result, this methodology indicates a value that is higher than the average maturity. However, as already mentioned, since this methodology does not consider intermediate coupon payments, it is not suitable for measuring the refinancing risk and should be applied only when comparing the Brazilian indicator to that of other countries.

$$AM = \frac{1}{PV} \times \sum_{i=1}^n PV_i \times M_i \quad (20)$$

where, M_i corresponds to the time between now and the bond maturity