

Part I

Chapter 3

Public debt sustainability

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1 Introduction

Public debt is a basic instrument for optimally distributing public policies over time. Through public indebtedness, the provision of public goods can be temporally disassociated from the taxation process required to fund it. For the debt instrument to fulfill its role, debt holders must believe that policies are such that contracts will be fulfilled. This requires fiscal policy to be sustainable.

This chapter focuses on three areas. First, it formalizes the idea of a sustainable fiscal policy. Second, it presents different evaluation measures that offer disciplined ways to assess whether a policy is sustainable. Last, it shows how the management of public debt plays a critical role in determining its sustainability.

A country's public debt is considered sustainable when government budget constraints can be met without disrupting its monetary and fiscal policies. As the chapter will clarify, this implies that the amount of the public debt should not exceed the present value of all future primary surpluses.

The concept of maintaining stable monetary and fiscal policy is crucial. Ultimately, government budget constraints can be met by acknowledging circumstances in which default on the debt (non-payment) or monetization will occur in ways that are consistent with mathematical expressions. In this case, the equations no longer represent true budget constraints but are rather pricing equations that answer the question about what is the current value of this debt which may not be paid.

Pricing equations allow two assets with identical contractual payment promises to have different market prices. Underlying the price difference is the perception that some contractual promises will not be kept. In contrast, an equation that represents a budget constraint must anticipate full payment in all periods and under all circumstances. Thus, the latter is the relevant concept for defining sustainability.

The relationship between sustainability and a government's fiscal balances gives the impression that the former can be determined in a way that is both objective and unambiguous. Unfortunately, that is not the case. In practice, it is impossible to know what future primary balances¹ will be, or the rate at which they will be discounted. Further, since a government's primary balance is a choice variable, any inferences with respect to future balances require expectations not only about the government's ability to generate the required surpluses but its willingness to produce them.

The remainder of this chapter is organized as follows. Following this Introduction, Section 2 makes the concepts more objective and Section 3 discusses practical ways to *evaluate* sustainability. Italics are necessary because, in general, no single measure can offer a definitive way to determine sustainability; rather, useful indicators for developing an educated view about the fiscal situation will be presented. Section 4 shifts the

¹ Fiscal savings generated by the government for debt payment. For details, see Part I, Chapter 1.

focus and discusses short-term aspects that can lead entities with perfectly sustainable debt management practices to insolvency. Section 5 addresses how a debt structure relates to its sustainability. Section 6 explains how public debt management in Brazil faces sustainability challenges, while Section 7 offers conclusions. Finally, the Appendix presents some debt sustainability and structure exercises performed by the Brazilian National Treasury.

2 Sustainability: formalization

To define debt sustainability in a very simple form, the world is considered deterministic, where all uncertainty regarding future behavior of relevant variables is disregarded. In this setting, a debt is sustainable when the present value of future revenue flows minus debtor expenses can pay for all that has been contractually agreed. However, even with this simplistic concept, some issues need to be clarified. First, the emphasis on values specified in the contract (or securities) is fundamental to avoid the circularity associated with the use of market value. Second, in most of what follows, debt will be considered in real terms, since the price level can be seen as a variable to adjust the debt value and create circularity similar to that related to the use of market value when discussing public debt.

2.1 A deterministic world

The inter-temporal government budget constraints is the set of all sequences of taxes and expenditures that guarantees that the contractual promises associated with the government's debt will be met. It is, however, useful to split the description of the budget constraints into two parts: the government flow constraint and the transversality condition. The combination of these two will then be shown to be equivalent to the present value representation.

For each period (in this case, a period corresponds to one year, since this is what is relevant from the budget standpoint), public debt evolves according to the following expression:

$$B_{t+1} = (1 + r_t)B_t + G_{t+1} - T_{t+1}, \quad (1)$$

where B_t is the value of government debt at moment t ; r_t is the interest rate in t ; T_t and G_t are government revenues and expenditures in t .

Obviously, this equation must hold to all periods. Therefore, in the following period:

$$B_{t+2} = (1 + r_{t+1})B_{t+1} + G_{t+2} - T_{t+2}.$$

Recursively substituting in (1) leads to,

$$B_{t+2} = (1 + r_{t+1})[(1 + r_t)B_t + G_{t+1} - T_{t+1}] + G_{t+2} - T_{t+2},$$

i.e.,

$$B_t = \frac{B_{t+2}}{(1 + r_{t+1})(1 + r_t)} + \frac{T_{t+2} - G_{t+2}}{(1 + r_{t+1})(1 + r_t)} + \frac{T_{t+1} - G_{t+1}}{(1 + r_t)}$$

The process can be continued to any $t+s$ time and express:

$$B_t = \frac{B_{t+s}}{\prod_{v=1}^s (1+r_{t+v-1})} + \sum_{v=0}^s \frac{T_{t+v} - G_{t+v}}{r_v} \quad (2)$$

It is important to define the variables: r_t expresses the interest rate of a security purchased in t , to be paid in $t+1$. Likewise, $G_t - T_t$ represents the primary deficit in t .

The constraint-flow (1) represents a minimum requirement, which could even be seen as an accounting identity. What makes the concept of sustainability interesting is the transversality condition. To understand it, the price in t of consumption in $t+s$ is given by

$$P_t = \left(\prod_{v=1}^s (1+r_{t+v-1}) \right)^{-1}$$

The transversality condition is, in this case,

$$\lim_{t \rightarrow \infty} P_t B_t \leq 0, \quad (3)$$

thus forcing the present debt value to approach a non-positive value when a sufficiently long time span is considered. This condition eliminates so-called Ponzi schemes, where a debt is always rolled over and never paid. In other words, it corresponds to the assumption that governments cannot live in a permanent state of indebtedness.²

If, on the one hand, the present value of government debt should not be positive, i.e., a government should not pay off its debt with more debt indefinitely, on the other it is only natural that the private sector cannot go into debt against the government indefinitely either. This justifies the imposition of a constraint (3) with an equal sign, in which case

$$B_t = \sum_{v=0}^{\infty} \frac{T_{t+v} - G_{t+v}}{\prod_{v=1}^s (1+r_{t+v-1})} \quad (4)$$

Imposing the transversality condition (3) with an equal sign ensures that the present value of primary surpluses will be the same as the debt value, thus showing the equivalence between the two representations.

In a world without uncertainties, the condition for public debt sustainability is precisely what can be expected: the Government, at some point, will raise enough money to cover its current expenses and also honor its commitments plus the corresponding interest.

² The underlying fact is that individuals (debt holders) are willing to postpone consumption, but not forever. It is interesting to acknowledge that, when prices fail to satisfy this condition, the outcome is called a bubble. The possibility of a bubbly asset cannot be discarded when there is uncertainty and individuals must form beliefs about the future behavior of prices. A large literature investigates conditions under which a bubble may arise and whether it is possible to empirically identify them. See Brunnermeier (2009) for a recent review.

In a world with uncertainties, however, the definition is not so simple. There are two fundamental differences with respect to the deterministic case. First, there are many possible trajectories for a government's primary surplus. How, then, should sustainability be defined? Should surpluses be sufficient to pay off the debt in all scenarios or only in the average? Second, while in the deterministic case, only one discount rate is defined (to avoid the emergence of arbitrage opportunities), in the stochastic case several rates of return are possible, according to their risk characteristics. Thus, the issue of what constitutes the relevant discount rate must be explored.

2.2 Uncertainty

Two dimensions exist where uncertainty matters. First, the value of government revenues and expenses and thus of primary surpluses is uncertain, hence there is uncertainty regarding the debt trajectory. Second, for each scenario, the *value* of accumulated surpluses or deficits may be different, depending on the rate at which they are discounted.

In the first case, if the surplus flow was variable but could be discounted at a rate that did not depend on the scenario³ (i.e., the value of a purchasing power unit is the same in all different scenarios), then the new sustainability condition would be one where the present value of government surpluses would be the same (equal to the debt value) for all scenarios discounted at this common rate.

Again, the issue is not simple. Given uncertainty, different assets with different risk features pay different dividends, a fact that makes the discount rate issue fundamental. Without uncertainty, there is only one rate – the risk-free rate. However, with uncertainty, there are several discount rates. Which is relevant?

The value of a purchasing power unit varies according to the state of the economy; this is the very essence of the risk concept. In difficult times (for example, during recessions), having an additional resource is much more valuable than in times of abundance. Thus, the surplus obtained during a recession is far more valuable than one secured in a period of abundance, for it is discounted at a lower rate. This rate, which varies according to the state of the economy, is the so-called risk-adjusted rate.

An intuitive way to understand government budget constraints in the presence of uncertainty is by imagining that at a given point in time, the government will decide or need to produce primary deficit, i.e., will choose to use more resources than what the money from taxes allows it to purchase. The question is: how will the government be able to finance the difference between the value collected and its total expenditures?

For individuals to temporarily and voluntarily relinquish purchasing power to the government, public authorities must promise that, when it is returned, purchasing power will be worth at least the same as it was when originally assigned to the government.

As payment will only be made in the future – and the future is uncertain – it has to be adjusted in temporal and risk dimensions. In the former, purchasing power will be worth less tomorrow than today. Thus, a positive interest rate must be paid. In the latter, risk-averse people are those who value income more when they have less of it. Thus, adjusting for risk requires that the government pay more, if it chooses, in states of nature (or scenarios) where the person's income is higher. It also implies that the government can afford to pay a little less in the states of nature where the person has no income at all.

³ The concept of history is formally used to define a sequence of random exogenous events affecting the problem's relevant variables. The word scenario will also be used to mean history, since this is a commonly used term.

The reasoning behind risk adjustment entails two consequences: (a) a fiscal policy in which surpluses are generated mainly in times of recession – i.e., when consumption is lower – is associated with lower-cost public debt; (b) the lower cost of debt stems from imposing higher taxes and/or reducing public goods/services when society is less capable of paying taxes and need more public goods and transfers. This is the great challenge of fiscal policy. If only the average cost of debt is considered, this might promote a socially perverse fiscal policy, where a decrease in the supply of public goods and an increase in the tax load occur precisely in times of recession.

Thus, although the issue of sustainability is emphasized here, policies which seem to make more sense with respect to reducing the financial cost of debt could be the most costly from the social perspective and thus politically unsustainable.

Nonetheless, even in a world marked by uncertainty, the condition of sustainability with real debt continues to be a present value calculation.⁴

$$B_t = E_t \left[\sum_{v=1}^{\infty} m_{t+v} [T_{t+v} - G_{t+v}] \right] \quad (5)$$

where $E_t[\]$ expresses the mathematical expectation conditioned on the information available at time t , and m_t denotes the so-called stochastic discount factor (or pricing kernel), a random variable that discounts future payoffs to incorporate the time and risk dimensions. In other words, the discount rate that is relevant for discounting surplus flows is risk-adjusted.⁵

It should be noted that this discussion is about real resources. For example, the private sector refrains from using certain resources today so as to somehow get them back in the future. Thus, the counterpart of a debt increase today is also in real resources and its true cost a situation of reduced public services in the future and/or increased taxes, with the latter including its deadweight costs.

2.2.1 Pricing equation versus budget constraint

Before moving to the next section, it is important to note that equation (5) is interpreted as a budget constraint rather than a pricing equation.

In the deterministic case, it was assumed in equation (5) that individuals cannot hold perpetual debts against the government,⁶ and B_t in (4) represents the present value discounted by the stochastic discount factor.

⁴ Bohn (1995) discusses the importance of correctly choosing the discount rate when building an economy where the fiscal policy rule ensures maintenance of the debt-to-GDP ratio, but debt is not sustainable (the transversality condition is violated) when the market interest rate is used as a discount criterion.

⁵ A good example occurs in the Consumption Capital Asset Pricing Model (CCAPM), where $m_{t+1} = u'(c_{t+1})/u'(c_t)$ is the intertemporal marginal substitution rate of the representative individual. Risk aversion causes $u'(c)$ to be decreasing in c , thus assigning a higher value to recession times, when consumption declines. Unfortunately, this model, devised by Breeden (1979) and Lucas (1978), is based on a very strong assumption of market completeness and has not been empirically successful. In a world without complete markets, the willingness to pay for consumption in different states of the world will differ across individuals, as will the measure of risk. While this form of heterogeneity makes the evaluation of sustainability more subtle, it may be important to improve the performance of the CCAPM. Indeed, several variables that incorporate heterogeneity in access to markets, utility functions where relative consumption is relevant, long-run risks have proven to be promising for developing an acceptable model for m_t .

When interpreted as a pricing equation, contractually established values should be replaced by those that will actually be paid. The latter will not coincide with the first whenever default, even if partial, occurs. In this case, there is no reason why contractually established values brought to present value should coincide with B_t . In other words, if discount rates are held constant, making B_t values and payment flows expected from the government compatible, this requires accepting that under certain circumstances, the values actually paid will differ from those contractually established.

Underlying the market value is, *inter alia*, the possibility of a state of insolvency. As a result, the lower the present value of expected surpluses, the lower the market value of public debt. In technical terms, equation (5) becomes a pricing equation rather than a constraint on possible surplus trajectories.⁷

The flip side of the above reasoning is that, if the value of the debt is lower than its contractual value discounted by that same rate, this means there are scenarios for which the value paid will be lower than the one contractually established, since future surpluses in pricing equations are discounted by the relevant rate in order to determine the debt's value. Default can occur in any of these scenarios.

The pricing equation nature of expression (5) is compounded when considering the nominal debt - here, the price level becomes an important debt-adjustment variable (Cochrane, 2005): Explicit default is replaced by price increase as a way to adjust the future surplus flow to the debt value established in contract.

If, however, equation (5) represents a budget constraint - and the contractually established values are set - then the equation shows which surplus trajectories are compatible with the contractual promises, i.e., those trajectories whose present value is equal to B_t which, in turn, is equal to the contractually determined flow discounted by the stochastic discount factor. It is only in the latter that government behavior is restricted, and only here that one can talk about sustainability.

3 Sustainability assessments

The concept of sustainability precludes definitions of objective measures to determine whether debt is sustainable. However, this chapter will present indicators that explore the future trajectory of surpluses, their related discount rates, and their compatibility with meeting government budget constraints.

3.1 Debt stationarity tests

One way to assess sustainability is based on public debt stationarity tests. This methodology, which gained momentum from the work of Hamilton and Flavin (1986), may be one of the most commonly used by researchers: In general, a stochastic process is stationary when it tends to revert to its average or to its trend following a random shock.

⁶ In this case, $\lim_{t \rightarrow \infty} E[m_t B_t] \geq 0$, combined with the no-Ponzi condition on the government (similar to (2)), gives us $\lim_{t \rightarrow \infty} E[m_t B_t] = 0$.

⁷ Mendoza and Oviedo (2003) posed the same question in a different way: "[...] The sustainability criterion assumed implicitly a mechanism for adjusting the fiscal accounts to meet the constraint, and failure to meet the criterion means failure to comply with that implicit mechanism." In other words, there is always a way to adjust the variables in order to ensure equality. Thus, the intent is to restrict possible adjustment mechanisms so as to ensure the equation will actually be a budget constraint and not a "way to determine the value of the debt."

In this case, the government's fiscal posture is such that following a shock which changes the value of the debt, surpluses are raised to force the debt to slowly return to its original value (or, if the debt shows a sustainable growth rate, surpluses are raised to the point at which debt growth returns to the previous rate. For example, if the debt has a rate of increase that is equal to the Gross Domestic Product (GDP) growth rate, surpluses are raised in such a way for the debt to revert to this trend). Here, the fiscal posture causes the value of the debt to comply with the transversality condition (3), i.e., that the debt be sustainable.⁸

In Brazil, this method was first used by Rocha (1997) and Issler and Lima (2000), who demonstrated that the hypothesis of stationarity for the Brazilian public debt between 1947 and 1992 cannot be rejected. The works also stated that adjustments are almost always obtained by raising taxes and that *seignorage* revenue needs to be added to tax revenue so it correspond to what is required by the expenditures in the long run. In other words, sustainability was maintained due to inflationary revenue. Ourives (2002) expands Issler and Lima's study (2000) to include quasi-fiscal deficits whereas Simonassi (2007) raises the possibility of structural breaks in the analysis.

Bohn (2007) questions this literature, claiming that an integrated debt of any order is sustainable.⁹ As stationarity cannot be tested for all orders, it is impossible, in practice, to reject sustainability (based on these tests). Thus, there is no way to prove that a debt is non-sustainable.

Another important criticism of stationarity tests is that they are performed with the observed time series: i.e., it is assumed the past is a reliable guide to the future. Even if researchers (and, for that matter, any theories) ultimately depend on "history" to project the future, stationarity tests are simplistic in that regard. Fundamental aspects of the historical evolution may be missed as they neglect very recent structural changes. Although purely statistical procedures can mitigate the problem (Simonassi, 2007) they cannot eliminate it.¹⁰

Alternatives to incorporating information not presented in the historical series are studies that seek to simulate debt dynamics through scenario-building approaches as a way to project the future. This could be an interesting and complementary way to evaluate situations where it is not thought that "the future will repeat the past," at least in a purely mechanistic form".

3.2 Trajectory of the debt-to-GDP ratio

In practice, a country's fiscal situation is usually determined by its debt-to-GDP ratio. There are many reasons to consider this measure an important solvency indicator. First, debt value *per se* means very little unless the size of the economy is known, since the value of potential surpluses depends, *inter alia*, on the overall resources an economy can produce. Also, every sustainability assessment requires assumptions about a country's capacity to make the sacrifices needed to generate surpluses which can ensure that equation (3) will be fulfilled. The real cost of this sacrifice depends on the wealth proportion to be used for this purpose.¹¹

⁸ Along this line, Bohn (1991) proposed testing to determine if the series of taxes and expenditures are co-integrated (with a (1,-1) co-integration vector). In other words, if the $G_t - T_t$ series is stationary.

⁹ If a series is stationary to begin with, i.e., B_t is stationary, it is called integrated of order zero; if their first differences are stationary, i.e., $B_{t+1} - B_t$ is stationary, it is called integrated of order 1; if differenced twice (i.e., taken the first difference of the first differences), it is called integrated of order 2 and so on.

¹⁰ Simonassi (2007) explores a model of multiple endogenous structural breaks to assess the sustainability of debt in Brazil from 1991-2006.

¹¹ Special mention should be made of the crucial difference between situations where debt is held by non-residents and those where the opposite is true. In the first case, the total debt and its charges represent a cost for residents. In the second, when distributive aspects are neglected, actual costs are only those associated with the so-called deadweight cost of taxation.

To express the dynamics of the debt-to-GDP ratio, the two sides of (1) are divided by Y_{t+1} ,

$$\frac{B_{t+1}}{Y_{t+1}} = (1 + r_t) \frac{B_t}{Y_t} \frac{Y_t}{Y_{t+1}} + \frac{G_{t+1}}{Y_{t+1}} - \frac{T_{t+1}}{Y_{t+1}}$$

i.e.,

$$b_{t+1} = \frac{1 + r_t}{1 + \gamma_t} b_t + g_{t+1} - \tau_{t+1}, \quad (6)$$

or also by

$$b_{t+1} - b_t = \frac{r_t - \gamma_t}{1 + \gamma_t} b_t + g_{t+1} - \tau_{t+1}, \quad (7)$$

where b , g e τ are, respectively, public debt, government expenditures, and tax collection as a proportion of GDP, and γ_t is GDP growth rate.

Throughout this discussion, the interest rate will be (at least on average) higher than the GDP growth rate. If both the GDP growth rate and interest rate are constant and the former is higher than the latter, then the first term on the right-hand side of the equation (7) is negative. This suggests that a government could experience deficits over its entire history, and debt as a percentage of GDP would still drop, permanently.

This is the practical reason why this possibility should be eliminated. However, there is also an economic reason: Situations where the economic growth rate exceeds that of the return on capital are cases of dynamic inefficiency (Blanchard et al., 1991). Basically, such an economy is one with excessive capital accumulation. In this case, consumption can increase without sacrificing the income available for future generations. In a dynamically inefficient economy, the government should, in terms of well-being, increase public debt to a point where pressure on the interest rate would halt the expansion of production and force capital accumulation to slow, driving the economy to an efficient path.

However, while there are policies that eliminate dynamic inefficiencies, this does not mean the inefficiencies do not exist. It is ultimately a purely empirical issue. With respect to the US economy over the last decade, the real interest rate of public debt securities did not reach an average of 1%, while the economic growth rate was over 3%. Does this mean the American economy is dynamically inefficient? Not necessarily.¹² In a world with uncertainty, the relevant rate of return issue is not as immediate. Indeed, alternative forms to verify dynamic efficiency have been proposed; the vast majority of these suggests that dynamic inefficiency, in the sense referred to above, does not characterize the capital accumulation process in the United States. In the case of Brazil, it appears no such studies exist.

Even where an economy is dynamically inefficient, with an average interest rate below the GDP growth rate, a question is whether this allows a government to build on this differential when conducting its debt trajectory.

¹² As previously noted, dynamic inefficiency is associated with excessive accumulation of capital. This seems to oppose the idea that US savings are below the optimal level. In addition, the average return for the physical capital stock in the US is much higher than the T-Bill rate, close to 10% a year.

The answer depends on the reasons for the low rate of return on public debt securities as compared to the GDP growth rate. Without precisely understanding its causes, it is impossible to say whether there is an opportunity to build on – through, for example, improving risk-sharing among individuals – or if the reduced interest rate reflects a high degree of risk-aversion, in which case there is no social gain in exploring the differential between the economic growth rate and interest rate. Unfortunately, given the current state of the art, there is no consensus for the low rate of return on American securities of recent years. Therefore, for the sake of simplicity, it will be assumed the rate of return is higher than the GDP growth rate, which is true in Brazil.

3.2.1 Calculating the surplus required for stabilizing the debt-to-GDP ratio

Most studies that use the debt-to-GDP ratio sustainability indicator explore this variable's behavior over time, to determine whether it shows a stable or downward trend.

In principle, fiscal policy sustainability indicators should result from the government's inter-temporal budget constraint, according to which the present value of taxes should be the same as that of expenditures, including interest on public debt and payment of the debt itself. How is stability of the debt-to-GDP ratio related to fulfilling (3)? If $r_t > y_t$, a stable debt in relation to GDP implies that the value of debt decreases over time. The stability of the debt/GDP ratio is therefore a *sufficient* condition to guarantee public debt sustainability.

The simplest use of the equation (7) to evaluate sustainability is to assess the surplus required to keep the debt-to-GDP ratio constant. In other words, supposing that $b_{t+1} = b_t = b$, the equation (7) will be

$$0 = \frac{r_t - \gamma_t}{1 + \gamma_t} b + g_{t+1} - \tau_{t+1} \Rightarrow \tau_{t+1} - g_{t+1} = \frac{r_t - \gamma_t}{1 + \gamma_t} b \quad (8)$$

The right side of the previous equation provides the surplus (as a proportion of GDP) required to stabilize the debt-to-GDP ratio in view of the current debt-to-GDP ratio, as well as the interest and growth rates of the economy.

For moderate GDP growth rates, the denominator of the expression on the right-hand side of (8) is of little relevance and an approximate estimate can be produced by using a very elementary method: For example, in a country whose debt-to-GDP ratio is 40% and where the cost of carrying (real rate) this debt is 7% while GDP grows by 5% a year, the surplus required to stabilize the debt-to-GDP ratio would be equal to

$$(r_t - y_t) \times b_t = (0,07 - 0,05) \times 0,4 = 0,008.$$

i.e., 0.8% of GDP.

Although simple, this method can be useful. Building tables for each hypothesis on the behavior of interest and GDP growth could serve as a basis, for example, for formulating the government's annual budget.

Keeping the debt-to-GDP ratio constant will ensure fiscal sustainability. However, forcing the primary surplus to satisfy this rule perpetually implies eliminating the fundamental role of public debt, which is to temporally disassociate public expenditures from debt financing by independently choosing the best time for producing one and the other. Therefore, forms to evaluate sustainability need to be presented that consider alternative trajectories of variables in (6) and do not necessarily imply keeping the debt/GDP ratio constant.

3.2.2 Tests based on the Value-at-Risk (VaR) methodology

Recent studies have used the adaptation of the Value-at-Risk (VaR) and Cost-at-Risk (CaR) risk management tools to assess debt sustainability,¹³ (see Barnhill and Kopits (2003), Bonomo et al. (2003), and Garcia and Rigobon (2004)). Several alternative trajectories for the debt/GDP ratio are generated from the stochastic version of the equation (1).

According to the VaR methodology, the highest value is evaluated in a way that the debt-to-GDP ratio does not exceed it at a predefined probability. In other words, first a level of confidence is chosen, which is generally 95%. A stochastic model generates alternative paths for all relevant variables and from them for the debt-to-GDP ratio. A future time period is set and the 95th percentile of the associated distribution of debt-to-GDP ratio is then found.

The CaR methodology is similar to that of the VaR and possibly more compatible with public debt management. First, while calculating the VaR requires that debt be marked to the market, the CaR methodology considers debt development as based on contractually established costs. Also, longer horizons are usually applied with the CaR methodology. Both favor the use of the second methodology for public debt management.

An important constraint in these studies is that determining the quantile (depending on the distribution share, deciles, percentiles, etc. can be evaluated) is a subjective act. Lima et al. (2008) use a quantile auto-regression approach, which combines stationarity tests (for the debt-to-GDP ratio) with the VaR methodology, in order to verify fiscal sustainability.

The first great advantage of this methodology is that instead of arbitrarily establishing a quantile to be considered "of risk," it identifies the critical quantile in which the debt trajectory switches from sustainable to unsustainable. It also incorporates non-linearities, which can produce interesting consequences in understanding how a government conducts its debt policy. When the debt-to-GDP ratio develops in a non-linear way, it might display moments of explosive behavior, while at the same time maintain a globally sustainable trajectory. However, this is not possible in a linear model, where local and global behaviors are identical.

Identifying the critical quantile helps determine the fraction of the time when the debt trajectory is sustainable as well as when it is explosive. The methodology also determines a tolerance limit for the fraction of time when the debt might be explosive, without implying it is non-sustainable, i.e., without the stochastic process being globally explosive.

An analysis of the Brazilian debt from 1976-2005 shows that, although it was above the critical quantile, i.e., it displayed an explosive behavior, debt was still globally sustainable 55% of the time: The tolerance limit was found by Lima et al. (2008) to be 60% of the time in the Brazilian case.

As the concept of sustainability is associated with that of stationarity, the methodology of Lima et al. (2008) is also subject to Bohn's (2007) criticism. Still, it seems an interesting way to provide information on the government's fiscal posture.

3.3 Government net assets and Asset and Liability Management (ALM)

If a company has total assets of R\$100 million and a debt of R\$50 million, in principle, its creditors believe that in a bankruptcy process, there will be enough assets to pay off the company's debt and thus feel

¹³ These methodologies are included in the set of instruments used by the Brazilian Treasury in the evaluation and strategic planning of public debt.

safe to grant new loans or roll over the existing debt. Ultimately, the firm will not need to enter a bankruptcy process, as it will find individuals willing to provide it with the necessary funds.

Using such reasoning, several countries have begun to survey their assets and liabilities to determine their state of insolvency. More important, they have applied measures that categorize their assets and liabilities with similar risk characteristics, which would reduce the volatility of the governments' net debt. This type of risk management, involving the administration of assets and liabilities, and known as Asset and Liability Management (ALM) has echoed in discussions about debt structure and set a benchmark in the way governments conduct their financial policies.

However, a government's main asset is its power to tax; its main liability is its obligation to provide public goods. Thus, any attempt to adapt the government's liability structure to its asset structure requires (a) good planning of how expenditures and collections will be distributed over time and (b) a debt structure that allows for the best planning.

4 Sustainability and solvency

This presentation considers that sustainability is associated with a government's capacity or willingness to honor its commitments, and that it has an infinite time horizon to do so. However, when analyzing the confidence crises many countries have experienced, it appears they usually arise not from fundamental changes in the way governments conduct their policies but rather from constraints in global liquidity. Such an observation seems to indicate that simply meeting inter-temporal constraints might not suffice. The reason global illiquidity and crises are related is that capital markets are not perfect, which affects most debt.

Along these lines, Xu and Ghezzi (2002) developed a methodology that identifies the probability of a country being unable to meet its financial commitments with other countries, allowing them to assess the likelihood of external debt crises due to global liquidity contractions. Baghdassarian et al. (2004) adapted the Xu and Ghezzi model to analyze total public debt and applied it to the Brazilian case.

By analyzing external debt, Xu and Ghezzi focused on the level of reserves and assessed the probability of a country running out of them at a given point in time.¹⁴ It is important to note that the emphasis in this chapter has now shifted from an analysis of sustainability to that of financial fragility. Both are important and the analysis of each should be seen as complementary rather than as a substitute for the one presented earlier.

Xu and Ghezzi's (2002) study refers to the fragility of external debt from the standpoint of scarce international reserves in a context of reduced international capital inflow. The same analysis applies to a country with high domestic debt concentrated in the short term, which Baghdassarian et al. (2004) explore. Under conditions of high domestic indebtedness, if the government does not have enough cash to pay its debt when the domestic market is volatile, the risk of default could increase significantly after successive failures to raise funds for this purpose; this could occur even when the debt is at a level that might be considered sustainable.

5 Sustainability and debt structure

Last, the sustainability of a country's public debt with respect to its characteristics could depend on the volatility of both the domestic and international financial markets as well as on the volatility of its economy.

¹⁴ In a world with perfect capital markets, reserves would be irrelevant since a country following a sustainable policy would always be able to raise money to cover its outstanding debt. However, capital market imperfections imply that perfectly solvent countries may not be able to raise money in times of global liquidity contractions.

For example, consider two countries with the same debt-to-GDP ratio at a given point in time and which, period-by-period, show the same expected behavior in the $r_t - \gamma_t$ difference. If neither country generates any surplus in the period, the country with the higher volatility in the $r_t - \gamma_t$ ratio will tend to present, at the end of the period, a debt-to-GDP ratio higher than that of the country with the lower volatility.

Based on additional hypothesis on the stochastic process of variables r and γ (taking r and γ as the instant rates corresponding to the initial formulation), the expected value of the debt-to-GDP ratio after a time interval of size s , $E_t[b_{t+s}]$, will be given by

$$E_t[b_{t+s}] = b_t \exp \left\{ \bar{r} - \bar{\gamma} + \frac{1}{2} \sigma^2 \right\}$$

where \bar{r} and $\bar{\gamma}$ are, respectively, the average interest rate and growth rate of the economy and σ^2 is the $r - \gamma$ variance. In other words, the more volatile this difference, the higher the expected debt-to-GDP ratio.

This ratio can be explained further by recalling that

$$\sigma^2 = var(r) + var(\gamma) - 2 \times cov(r, \gamma),$$

where $var(.)$ expresses the variance of a variable and $cov(.,.)$, the co-variance between two variables. In this case, the variance of $r - \gamma$ depends both on the sum of the variance of the rate of return and the variance of the growth rate of the economy, and, as well as, on the co-variance between these two variables.

Controlling the volatility of the GDP growth rate is something far beyond what debt managers can attempt to achieve. Volatility of the debt-carrying rate, however, can be reduced by efficient public debt management: Long-term, fixed debts (or price linked) for example, have a more or less constant carrying rate, which means volatility of r can be kept at acceptable levels. But managers can follow an even more interesting strategy: They can choose a debt structure through which the carrying rate strongly co-varies with the GDP growth rate. At the same expected cost, this type of public debt design ensures a lower expected growth of the debt-to-GDP ratio.

An appropriate question, therefore, is if this type of design is possible at a given cost. The issue is that an asset that co-varies positively with GDP growth rate is a risky asset for savers (as opposed to an asset that co-varies negatively and thus offers hedging possibilities). Savers will demand a risk premium to carry a debt with high GDP correlation by raising \bar{r} ¹⁵ The optimal debt design should take all such effects into account.

It has been assumed that surplus in the period is equal to zero. In practice, however, the timing of surpluses is another choice variable that can be used to prevent volatility of the growth rate of the debt-to-GDP ratio from being high. In fact, there is an important reason why this instrument might be attractive.

¹⁵ In a world of complete markets and perfectly rational agents, the increase in \bar{r} is such that the debt management policy becomes irrelevant, in the sense that savings of resources occur through an increase in the risk associated with the debt structure. Obviously, markets are not complete and agents are not endowed with the unlimited rationality required for irrelevance to apply. The importance of this consideration lies more in showing that apparent savings of resources could occur at the expense of a rise in the associated risk, especially in the form of a demand to deviate the temporal distribution of taxes and/or more efficient public expenditures from the standpoint of social well-being. Also important is the fact that a full answer to the question of the optimal debt structure will ultimately depend on the use of structural models for the underlying economy.

If a country fails to sufficiently increase its primary surplus during a crisis of confidence, the perception that public debt is not sustainable could result in higher interest rates, which in turn could push the government into a vicious cycle where higher debt leads to an increase in its own carrying costs. This non-linearity in debt behavior exacerbates the value of a stable structure.

Thus, excessive volatility should be avoided through a debt structure whose dynamic is not too sensitive to the short-term movements of relevant variables and through fiscal behavior aimed as much as possible at neutralizing these adverse events.

Favero and Giavazzi (2007) also suggest that countries characterized by a less stable economic environment should consider that the conditions required for stabilizing the debt are more demanding than those for countries with a steadier environment. In particular, a country's solvency is subject to closer scrutiny where economies are more volatile. A policy that seeks to generate greater stability increases the creditors' perception of the government's fiscal responsibility and, consequently, reduces the adverse effects of these non-linearities in debt behavior.

Finally, as noted in Section 4, many events that lead to a repudiation of the countries' debts are not associated with an abnormal increase in the debt-to-GDP ratio; rather they are connected with a liquidity crisis in financial markets that prevents countries that were maintaining some stability in their debt-to-GDP ratio from refinancing their debt. Therefore, to avoid vulnerability in such times, a government should eschew short-term debts or those whose maturity is far too concentrated, for they expose a government to a risk that is not only excessive but, in the case of concentration, unjustifiable.

6 Sustainability and public debt management in Brazil

In Brazil, the Finance Ministry continually analyzes public debt sustainability. To that effect, the Economic Policy Secretariat (SPE) and the National Treasury carry out different, although complementary, evaluations. While SPE evaluates the dynamic of the debt-to-GDP ratio in light of the formalization mentioned in equation (6), to better define the primary surplus required to ensure fiscal solvency¹⁶ and ultimately long-term macroeconomic equilibrium, the National Treasury incorporates elements of sensitivity into this analysis, in light of governmental assets and liabilities.

As discussed in the previous section, public debt managers can contribute to sensitivity analyses by adding the profile of the current public debt and the financing strategy for the coming years. They can also add the risk dimension and refine the evaluation of the costs of assets and liabilities to this same analysis, given their expertise in managing debt risks.

In this sense, the National Treasury's public debt managers developed tools to analyze debt sustainability: For data, they use the primary surplus for the following years, as established in the Budgetary Guidelines Law (LDO) and add expectations with respect to the real interest rate and economic growth, and the debt financing strategy. Uncertainties associated with both the debt and the variables included in the analysis are also considered. These include studies on the optimal public debt profile, or benchmarks, that seek to measure the cost and risk of different profiles in terms of their impacts on the debt-to-GDP ratio.¹⁷

¹⁶ According to a proposal by the Ministries of Finance and Planning, the Federal government would submit to the National Congress annually the Budgetary Guidelines Law (LDO), which will guide the budget for the following year. In its annex on fiscal targets and based on SPE parameters, the law establishes the government's primary surplus target for the three subsequent years as well as its expectation regarding evolution of the public debt vis-à-vis the projected macroeconomic parameters.

¹⁷ For details on the optimal public debt composition model, see Part II, Chapter 3.

In this regard, debt structure has an important effect on fiscal sustainability. First, higher outstanding debt volatility leads to increased financing costs for the longer term. Second, there are the non-linearities described by Fávero and Giavazzi (2006), in which temporary changes in debt trajectory can be seen as permanent and thus raise costs and ultimately exacerbate volatility, with the already-mentioned consequences for debt carrying costs.

In view of the debt structure-sustainability ratio, debt management becomes an important component in a nation's fiscal policy. For this reason, the National Treasury has, for over a decade, improved the debt evaluation and planning instruments. The choice of appropriate instruments has increasingly taken into account not only immediate costs but also the risks involved in a balanced profile. Descriptive statistics, exercises using the Cost-at-Risk (CaR) methodology for outstanding debt (financial risk) and Cash-Flow-at-Risk (CFaR) for future flows (refinancing risk), among others, are produced to evaluate the current situation as well as formulate long-term guidelines through the development of optimal profiles (benchmarks) and transition strategies.¹⁸

With regard to refinancing risks, Brazilian public debt managers have attempted to "smooth" securities' maturity. Even when interest rates are highly volatile, the use of floating-rate securities has allowed the government to separate the interest rate risk from the refinancing risk, thus allowing it to carry the first risk (whose effects are fundamentally associated with sustainability) as it eliminated, or sharply reduced, the second.¹⁹

7 Conclusion

Although the concept of sustainability can be formalized in a way that is free of ambiguities, no test exists that can clearly indicate whether the debt trajectory of a country is sustainable. In practice, debt sustainability, in addition to involving great uncertainty about the behavior of variables that are difficult to foresee, depends on political options whose evaluation involves developing concepts about the posture of present and future governments.

Still, several indicators examined here could be of great use in assessing, with some discipline, the information contained in historical series. All the indicators assume, somehow, that the past is a good guide for understanding the future (which is what really matters from the standpoint of sustainability) in view of the way fiscal policy is conducted. Institutions change, governments change, and the future need not repeat the past. Thus, it is important that technical evaluations not be isolated but rather understood as a consistent way to organize some of the information offered by history.

It is also important to note that although the concept of sustainability is associated with long-term fiscal policy, many government insolvency crises are characterized by short-term liquidity constraints. Thus, an analysis of a government's fiscal situation should include both dimensions of the issue.

Finally, the government's fiscal posture and public debt structure should not display a trajectory that could be seen as unsustainable, mainly because this can affect the cost of debt roll-over and ultimately make non-sustainability a self-fulfilling prophecy.

¹⁸ The appendix to this chapter provides an example of a study carried out within the National Treasury, for merely illustrative purposes. More information on these methodologies as well as on how they affect public securities emission strategies will be provided in Part II, Chapters 2 and 3.

¹⁹ This effect is demonstrated by the securities' characteristic of having high average maturity, simultaneously with minimum duration.

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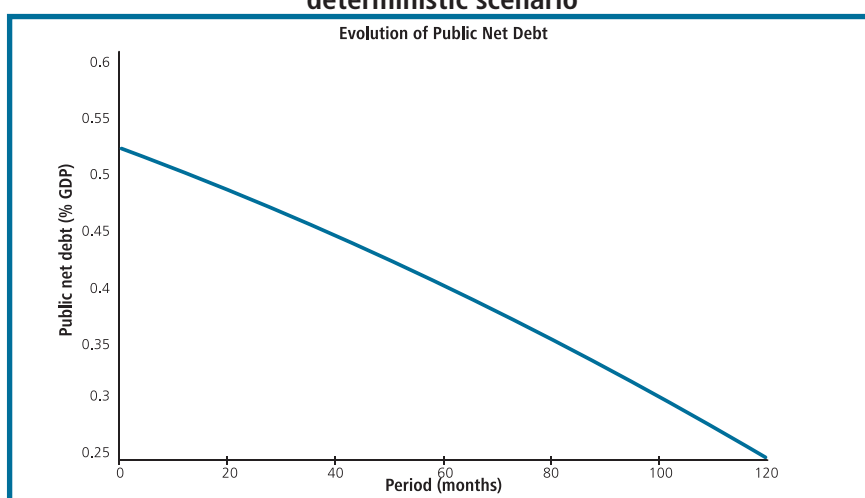
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Appendix

The exercise presented in this Appendix illustrates the type of sustainability analysis that can be performed when introducing the uncertainty aspect and public debt profile. It is intended for illustrative purposes alone and reflects the market perception of the behavior of some key-variables in the recent past. Similar exercises will be explored in Part II, Chapter 3.

The exercise considers some basic premises for the chief determinants of the debt dynamic for a 10-year period: nominal interest rates, inflation, GDP growth, and primary surplus.²⁰ A first sustainability test is performed in the basic deterministic scenario. Graph 1 shows the trajectory expected for the debt-to-GDP ratio.

Graph 1. Trajectory expected for the debt/GDP ratio - deterministic scenario



Stochastic trajectories are then generated for all variables. At each point in time, it is possible to obtain a distribution of debt-to-GDP ratios. The parameters of underlying stochastic processes have been chosen in such a way as to produce an average value similar to that of the deterministic scenario.

Alternatively, the parameters can be chosen in a way that would produce the deterministic scenario when volatility is "neutralized," to illustrate its impact on the average debt-carrying cost

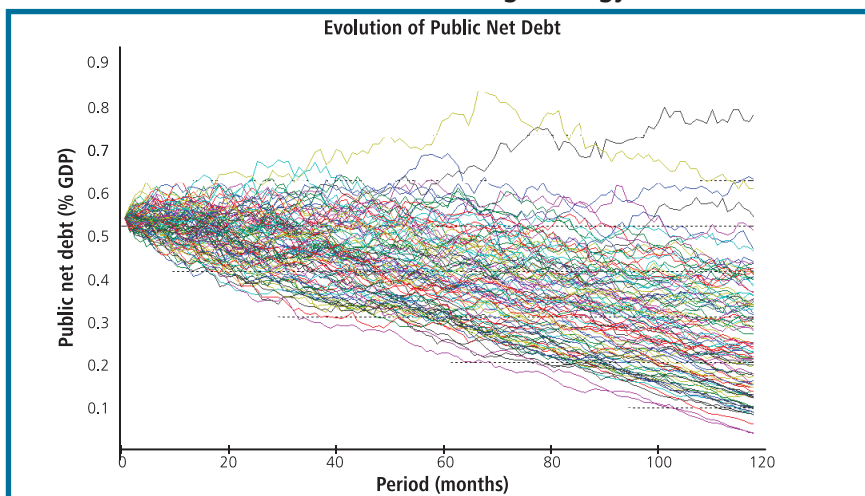
The exercises also include an explicit hypothesis on the financing strategy. If a government chooses to lengthen the debt term by either extending the maturity of fixed securities or reducing the share of post-fixed securities, variations in the interest rate will affect a smaller public debt percentage, thereby reducing its carrying cost volatility. This strategy involves an additional cost, but a rationale such as this can be formally incorporated and, under some explicit hypothesis, quantified.

These exercises provide the government with a powerful tool when it designs its long-term debt strategies.

Graph 3 illustrates an exercise similar to that in Graph 2, as it includes an explicit hypothesis on the debt refinancing strategy

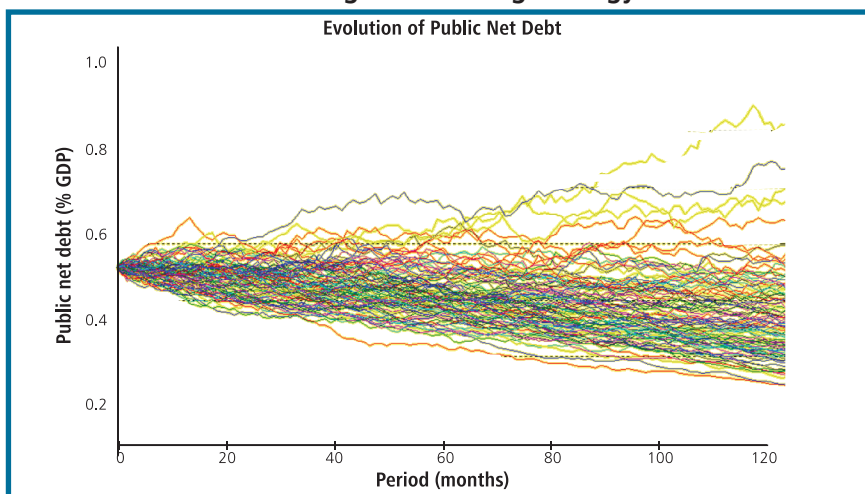
²⁰ For details on comparable simulations, see Part II, Chapter 3.

Graph 2. Distribution of debt-to-GDP ratios within different horizons without a refinancing strategy



While debt is composed 100% of floating rate instruments in the first exercise, the refinancing strategy involves 10-year fixed instruments.²¹ Its higher cost, which is due to the risk premium of the interest rate charged by creditors, is offset by lower debt vulnerability to changes in interest rates.

Graph 3. Distribution of debt-to-GDP ratios within different horizons, including a refinancing strategy



²¹ The refinancing strategy assumes that 1% of the floating share falls due and is exchanged on a monthly basis for 10-year fixed instruments. At the end of this period, 100% of the debt is in fixed instruments.

