Methodological Note Number 1 – Federal Public Debt Indicators

December 2023

1. Introduction

This Methodological Note aims to present the concept and the calculation methods of the main Federal Public Debt – FPD indicators, such as outstanding debt, outstanding profile by adjustment factor, debt maturity within 12 months, average maturity, average maturity of Domestic FPD issuances, average term to maturity, monthly average cost of outstanding debt, cumulative 12-month average cost of outstanding debt and average cost of DFPD issuances and liquidity index.

For the purposes of this methodological note:

I – Federal Public Debt – FPD is the debt amount incurred by the National Treasury, comprising of the Domestic Federal Public Securities Debt - DFPSD and the External Federal Public Debt - EFPD;

II – EFPD is the sum of the External Federal Public Securities Debt - EFPSD and the External Federal Contract Debt; and

III – Federal Public Securities Debt– FPSD comprises the sum of the Domestic Federal Public Securities Debt - DFPSD and the External Federal Public Securities Debt – EFPSD..

2. Main FPD indicators

2.1. Outstanding FPD

Outstanding FPD is the sum of FPSD (domestic and external) and EFPCD.

The FPSD outstanding on a given date is the sum of the financial amount of every bond on that date. The financial amount of a specific security is the result of its average unit price multiplied by its outstanding quantity, represented by the following formula:

$$\text{Outstanding} = \text{UP} \cdot \text{Q}$$

where,

UP: average unit price of the security, which represents the weighted average of the prices of all outstanding issuances related to this security;

Q: outstanding quantity of the related security on the reference date.

a) the average unit price of the security is the product of the price of each issuance, on the reference date, weighted by the respective quantity issued, according to the formula:

$$\text{UP}_{\text{security}} = \frac{\sum_{\text{issuance}=1}^{\text{n}} \text{UP}_{\text{issuance}} \cdot \text{Q}_{\text{issuance}}}{\sum_{\text{issuance}=1}^{\text{n}} \text{Q}_{\text{issuance}}}$$

where,

$\mathrm{UP}_{\mathrm{issuance}}$: UP of each issuance, on the reference date; and

$\mathrm{Q}_{\mathrm{issuance}}$: Quantity of securities issued in each issuance.

b) the price related to each issuance of a given security, on the outstanding reference date, is the product of its Updated Nominal Value – UNV and its quote, according to the formula:

$$\mathrm{Price} = \mathrm{UNV} \,.\, \mathrm{quote}$$

c) for securities that present a flow of amortization payments before maturity, quotes always consider the last outstanding balance and the amortized UNV;

d) the UNV of a given security represents its nominal value on its reference date or, for securities without a reference date, on its date of issuance or most recent amortization, updated by the variation of its adjustment factor to the outstanding reference date, according to the following formula:

$$\mathrm{UNV} = (\mathrm{nominal\ value}) \,.\, (1 + \Delta\mathrm{index})$$

where,

$\Delta\mathrm{index}$: adjustment factor or currency variations related to the security, over the specified period.

e) for fixed rate securities, the UNV is always R$ 1,000.00 (one thousand Brazilian reais);

f) the quote for each issuance is the sum of the present value of each future flow of principal and interest coupons, using the internal rate of return - IRR at its issuance as the discount rate (effective interest methodology), according to the formula:

$$quote = \sum_{\mathrm{t=0}}^{\mathrm{n}} \frac{\mathrm{Ft}}{(1 + \mathrm{i})^{\mathrm{t}}}$$

where,

Ft: flow of principal and interest coupons of the outstanding security, as a percentage of its face value;

t: fraction of the year between the quote's reference date and the maturity date of the specific flow. For domestic securities, this is calculated using the number of business days/252 standard. For external securities, the 30/360 standard (for Globals) and actual/actual standard (for EuroBonds) are adopted; and

i: IRR at issuance.

g) in relation to External FPD securities, all calculations are made using the original currency, and the result converted to Brazilian currency using its selling rate at outstanding reference date.

The External Federal Public Contractual Debt outstanding is the sum of the outstanding balance of each contract, with accrued interest, converted from the original currency to Brazilian currency using the selling rate at the outstanding reference date.

2.2. Outstanding Profile by Adjustment Factor

The Outstanding Profile by Adjustment Factor is the breakdown of the FPD outstanding by the different adjustment factors applicable to the different securities and indicates the relative share of each of the following adjustment categories in the outstanding debt: fixed-rate, consumer price-index (inflation-indexed), floating-rate (securities with varying interest rates) and exchange-rate (foreign currency-denominated or referenced debt).

The share of each category is obtained by dividing its outstanding financial amount by total DPF outstanding.

2.3. 12-Month Maturity Share

The share of outstanding debt maturing within 12 months results from dividing the sum of principal and interest payable on securities and contracts during the 12 months subsequent to the reference date by the total FPD outstanding on that date, according to the formula:

$$12 - \text{Month Maturity Share} = \frac{\sum_{t=0}^{12} Ft \,.\, (1 + i)^{-t}}{\sum_{t=0}^{n} Ft \,.\, (1 + i)^{-t}}$$

where,

Ft: Maturing principal and interest flow from securities/contracts, as a percentage of face value;

t: fraction of the year between the reference date and the maturity date of the flow. For Domestic securities, this is calculated using the number of business days/252 standard. For external securities, the 30/360 standard (for Globals) and actual/actual standard (for EuroBonds) are adopted; and

i: Average IRR of the security/contract on the reference date, as a per annum percentage

Since it compares the flow of maturities in relation to the outstanding debt, the methodology takes into account the present value of the maturing flow, discounted by the average IRR of each security/contract. The average IRR of a security represents the average rate of return of all issuances related to this specific security, and is obtained, on an annual basis, using an iterative method from the average price of each security and the maturity of its principal and interest flows.

2.4. Average Maturity

The FPD average maturity is calculated from the average maturities of its securities and contracts, weighted by their respective outstanding amount on the reference date.

The average maturity of FDP securities and contracts represents the average maturity of future principal and interests flows, weighted by their respective present values, according to the formula:

$$\text{Average Maturity} = \frac{\sum_{t=0}^{n} t \,.\, Ft \,.\, (1+i)^{-t}}{\sum_{t=0}^{n} Ft \,.\, (1+i)^{-t}}$$

where,

Ft: Principal and interest flow from maturing securities/contracts, as a percentage of face value;

t: fraction of the year between the reference date and the maturity date of the respective flow. For Domestic securities, this is calculated using the number of business days/252 standard. For External securities and contracts, the 30/360 standard (Globals) and actual/actual standard (EuroBonds) are used; and

i: Average IRR of the security/contract on the reference date, as per annum percentage

2.5 - Average Maturity of DFPSD Issuances

The average maturity of DFPSD issuances is calculated from the average of the maturities of each issuance, weighted by the respective financial amounts issued. The maturity of each issuance is calculated according to the formula:

$$\text{Maturity}_{\text{issuance}}\,(\text{duration}) = \frac{\sum_{t=0}^{n} t \,.\, Ft \,.\, (1+i)^{-t}}{\sum_{t=0}^{n} Ft \,.\, (1+i)^{-t}}$$

where,

Ft: principal and interest flow from the maturing security, as a percentage of face value;

t: fraction of the year between the issuance date and the maturity date or coupon payment date of the security, using the number of business days/252 as standard.

i: issuance IRR, as a per annum percentage.

The issuances included in the calculation of this indicator are those made at public offerings to the Market and Tesouro Direto portfolios.

2.6. Average Term to Maturity – ATM

The average term to maturity – ATM is the indicator of maturity that represents the weighted average term of the flow of principal only, related to DPF securities and contracts, considering their face values, according to the formula:

$$\mathrm{ATM} = \frac{\sum_{t=0}^{n} t \,.\, \mathrm{FPt}}{\sum_{t=0}^{n} \mathrm{FPt}}$$

where,

Ft: Maturing principal flow from the securities/contacts; and

t: fraction of the year between the reference date and the maturity date of the respective flow. For Domestic securities, this is calculated using the number of business days/252 standard. For External securities/contracts, the 30/360 standard (Globals and contracts) and actual/actual standard (EuroBonds) are adopted.

2.7. Monthly Average Cost

The monthly average cost of the outstanding FPD is the indicator that represents the carrying cost of the FPD in the reference month. It is calculated through the average cost of each FPD security and contract, weighted by its respective outstanding amount on the reference date, according to the formula:

$$\mathrm{TMC}_{\%\mathrm{pa}} = \frac{\sum_{b=1}^{n} \mathrm{MC}_{b\%\mathrm{pa}} \,.\, \mathrm{FA}_{b}}{\sum_{i=1}^{n} \mathrm{FA}_{b}}$$

where,

$\mathrm{TMC}_{\%pa}$: FPD monthly average cost, as a per annum percentage;

$\mathrm{MC}_{\%pa}$: monthly average cost of each security/contract (*b*), as a per annum percentage; and

FA: Financial amount of each security/contract (*b*), consisting of its outstanding stock as of the last day of the month prior to the reference month.

The monthly average cost of each security/contract is calculated according to the formula:

$$\mathrm{MC}_{\%\mathrm{pm}} = [(1 + \mathrm{IRR}) \,.\, (1 + \Delta\mathrm{index})] - 1$$

where:

$\mathrm{MC}_{\%pm}$: monthly average cost of each security/contract, as a per month percentage;

IRR: internal rate of return of the security/contract, as a per month percentage; and

$\Delta\mathrm{index}$: adjustment factor or currency variations applicable to the security/contract, over the reference month.

In order to normalize and eliminate the effects of different numbers of business days across months, the monthly average cost is calculated on a per annum basis, according to the formula:

$$\mathrm{MC}_{\%\mathrm{pa}} = \left(1 + \mathrm{MC}_{\%\mathrm{pm}}\right)^{\frac{252}{\mathrm{bd}}} - 1$$

where:

$\mathrm{MC}_{\%\mathrm{pa}}$: monthly average cost, as a per annum percentage;

bd: number of business days in the reference month in order to calculate the average cost of outstanding DFPSD. For EFPD securities and contracts, the day count standard of each financial instrument is adopted.

2.8. Cumulative 12-Month Average Cost

The cumulative 12-month average cost of the FPD is the indicator that presents, as a per annum percentage, the carrying cost of the public debt during the previous 12 months. The calculation of this indicator, for both securities/contracts and their groupings, is as follows:

$$\mathrm{CAC}_{\mathrm{t}} = \prod_{\mathrm{k=t-11}}^{\mathrm{t}} \left(1 + \mathrm{MC}_{\mathrm{k}}\right) - 1$$

where,

$\mathrm{CAC}_{\mathrm{t}}$: cumulative 12-month average cost of securities/contracts, in month *t*, as a per annum percentage; and

MC_{k}: monthly average cost of securities/contracts, in month *k*, as a per month percentage.

2.9. DFPD Average Cost of Issuances

The average cost of DFPSD issuances is calculated by the average of each security's average cost of issuances, weighted by its respective financial amount issued in the previous 12 months. The average cost of issuances of each security is calculated as follows:

$$\mathrm{ACI} = \left[\left(1 + \text{Average } \mathrm{IRR}_{\mathrm{issuances}}\right) . \left(1 + \Delta\mathrm{Index}_{\%\mathrm{py}}\right)\right] - 1$$

where,

$\text{Average IRR}_{\text{issuances}}$: average IRR of the issuances of each security in the previous 12 months, as a per annum percentage; and

$\Delta\text{Index}_{\%\text{pa}}$: average daily variation of the adjustment fator, as applicable to each security, as a per annum percentage.

The average IRR of the issuances of each security in the previous 12 months, as a per annum percentage, is calculated as the average of the IRR at issuance, considering issuances at public offerings made in the 12 months prior to the indicator reference date.

$$\text{Average IRR}_{\text{issuances}} = \frac{\sum_{\text{issuance}=1}^{\text{n}} \text{IRR}_{\text{issuance}} \cdot \text{FV}_{\text{issuance}}}{\sum_{\text{issuance}=1}^{\text{n}} \text{FV}_{\text{issuance}}}$$

where,

$\text{IRR}_{\text{issuance}}$: IRR of each issuance, as a per annum percentage; and

$\text{FV}_{\text{issuance}}$: financial amount of each issuance.

The formula for the average daily variation of the adjustment fator applicable to each security is as follows:

$$\Delta\text{Index}_{\text{security\%pd}} = \frac{\sum_{\text{issuance}=1}^{\text{n}} \Delta\text{Index}_{\text{issuance\%pd}} \cdot \text{FV}_{\text{issuance}} \cdot \text{bd}_{\text{issuance}}}{\sum_{\text{issuance}=1}^{\text{n}} \text{FV}_{\text{issuance}} \cdot \text{bd}_{\text{issuance}}}$$

where,

$\Delta\text{Index}_{\text{security\%pd}}$: average daily variation in the adjustment factor applicable to each issuance during the 12 months prior to the indicator reference date;

$\text{FV}_{\text{issuance}}$: financial amount of each issuance; and

$\text{bd}_{\text{issuance}}$: number of business days between the date of each issuance and the indicator reference date.

a) the daily variation in the adjustment factor applicable to each issuance should be calculated between the date of issuance and the indicator reference date, in order to capture all of the available variation, according to the formula:

$$\Delta\text{Index}_{\text{issuance\%pd}} = (1 + \Delta\text{Index}_{\text{issuance}})^{\left(1/\text{bd}_{\text{issuance}}\right)} - 1$$

b) the average daily variation in the adjustment factor applicable to each security should be converted to an annual rate using the following formula:

$$\Delta\text{Index}_{\text{security\%pa}} = (1 + \Delta\text{Index}_{\text{security\%pd}})^{(252)} - 1$$

The issuances included in the calculation of this indicator are those made at public offerings to the Market and Tesouro Direto portfolios.

2.10. Liquidity Index

The liquidity index informs the sufficiency, in months, of the liquidity reserve to cover Domestic Federal Public Debt (DFPD) payments. The liquidity reserve consists of the resources, in Brazilian reais, held in the Union's Single Account with the Central Bank of Brazil and earmarked for the payment of said debt. The liquidity index is calculated as follows:

$$\text{LI} = \text{n} + \left[\left(\text{LR}_0 - \sum_{i=1}^{n} FV_i \right) \Big/ FV_{n+1} \right]$$

where,

n: number of months following the reference date for which the liquidity reserve fully covers the respective maturity flows;

LR_0: sum of cash availabilities earmarked for domestic debt payment and cash balance from proceeds of government bond issuances on the last day of the reference month (month 0).

FV_i: Flow of the projected maturities of month i, at current value, taking into account outstanding securities in month 0 and a specific scenario for all adjustment factors. Maturities both of principal and interest of securities held by the public, and interest payments on securities held by the Central Bank are included.